BUNCH SHAPE IN CYCLIC ACCELERATORS

UNIVERSITY OF BIRMINGHAM

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STUDIES OF BUNCH SHAPE IN CYCLIC ACCELERATORS

BY

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A THESIS
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FOREWORD

In the fall of 1948, the National Research Council of Canada awarded to the writer a Special Scholarship. This gave him the opportunity of spending the three years 1948 - 1951 at Birmingham University, where the work described in the following chapters was done. This period was spent in studies relating to the two cyclic accelerators at the University. One of these is a proton synchroton, the first such machine to be designed; the other is a fixed frequency cyclotron which has reached the highest particle energy attained in a machine of this type.

Chapter I indicates the origin and development of the various types of cyclic accelerators and Chapter II gives a brief description of the principles underlying their operation. Chapter III is concerned with producing the optimum bunch shape after injection in the synchrotron. A practical method is developed for realizing the well-known advantages of avoiding betatron oscillations by insuring that protons have, throughout the injection period, the energy appropriate to the equilibrium orbit. There were two parts to this problem. One was the production of a voltage rising at the rate of 90 megavolts per second throughout the injection period. The other was the stabilization of a 500 K.V. source to a high accuracy. In Chapter IV some of the demands placed on the extraction system by the bunch shape of the proton beam are considered. It is shown that existing techniques are adequate for an extraction system capable of removing the beam in one revolution but that gradual removal of the beam is not feasible.

In Chapter V devices are discussed for obtaining a detailed knowledge of the dimensions and position of the proton bunch during acceleration in the synchrotron. Measurements have been made on electrodes of Brookhaven design and an improved form of electrode has been arrived at. These electrodes have been tested using bunches of electrons to simulate the proton beam. Circuits have been obtained suitable for computing the beam coordinates throughout the accelerating cycle and presenting them as a function of time on a long-persistence cathode ray tube. A method of presenting beam phase and width throughout the cycle is outlined.

In Chapter VI a method of measuring bunch shapes in the cyclotron is cyclotron is described. Measurements on the Birmingham University fixed frequency cyclotron are discussed in the light of an approximate expression for the phase of the particle during acceleration as a function of the orbit radius, input phase, magnetic field, and dee voltage. These measurements show that the cyclotron can produce bunches of particles occupying about 20 electrical degrees or 5.5×10^{-9} seconds. The phase of the ions is shown to provide a suitable basis for an automatic tuning device.

This work was begun under the direction of Professor M.L. Oliphant and completed under the supervision of Professor P.B. Moon whom the writer wishes to thank especially for his stimulating interest and encouragement. The writer wishes also to express his appreciation to his associates for their cooperation, especially to Dr. L. Riddiford without whose assistance the work described in section 5.23 could not have been undertaken.

CHAPTER I

HISTORICAL NOTE

1.1. The search for methods of accelerating particles probably began upon the discovery of the interesting experiments which could be done with the high speed particles given off by natural radio active substances. The most straightforward method of accelerating charged particles is to allow them to move under attractive forces between two regions of different potential. Methods of producing and maintaining the necessary difference of potential are still being improved.

1.2. The earliest of these is probably that described by Moseley[1] in 1913. He used the beta emission of radium to build up a charge of 150 K.V. on an isolated sphere in vacuum. It is interesting to note that this method has been suggested again[2] as recently as 1947. Much improved performance is now possible because of the lowered cost of radioactive material produced in piles and the better understanding of high vacuum insulation.

1.3. In the years during which disintegration experiments were carried out using natural radioactive materials, the importance of experiments with high speed particles became more and more widely recognized, and the limitations of natural particles were more keenly felt. Thus about ten years after the first disintegration in 1919 a great number of methods of accelerating particles were approaching usable

form. The Gamow theory, which showed that very high energies were not necessary to produce disintegrations, gave an additional impetus and some three or four years later, in 1932, one of the machines[3] actually produced disintegrations.

1.4. Figure 1.4, the graph of accelerator publications per year versus time, shows how sharpky marked was the beginning of this activity and how rapidly it increased.

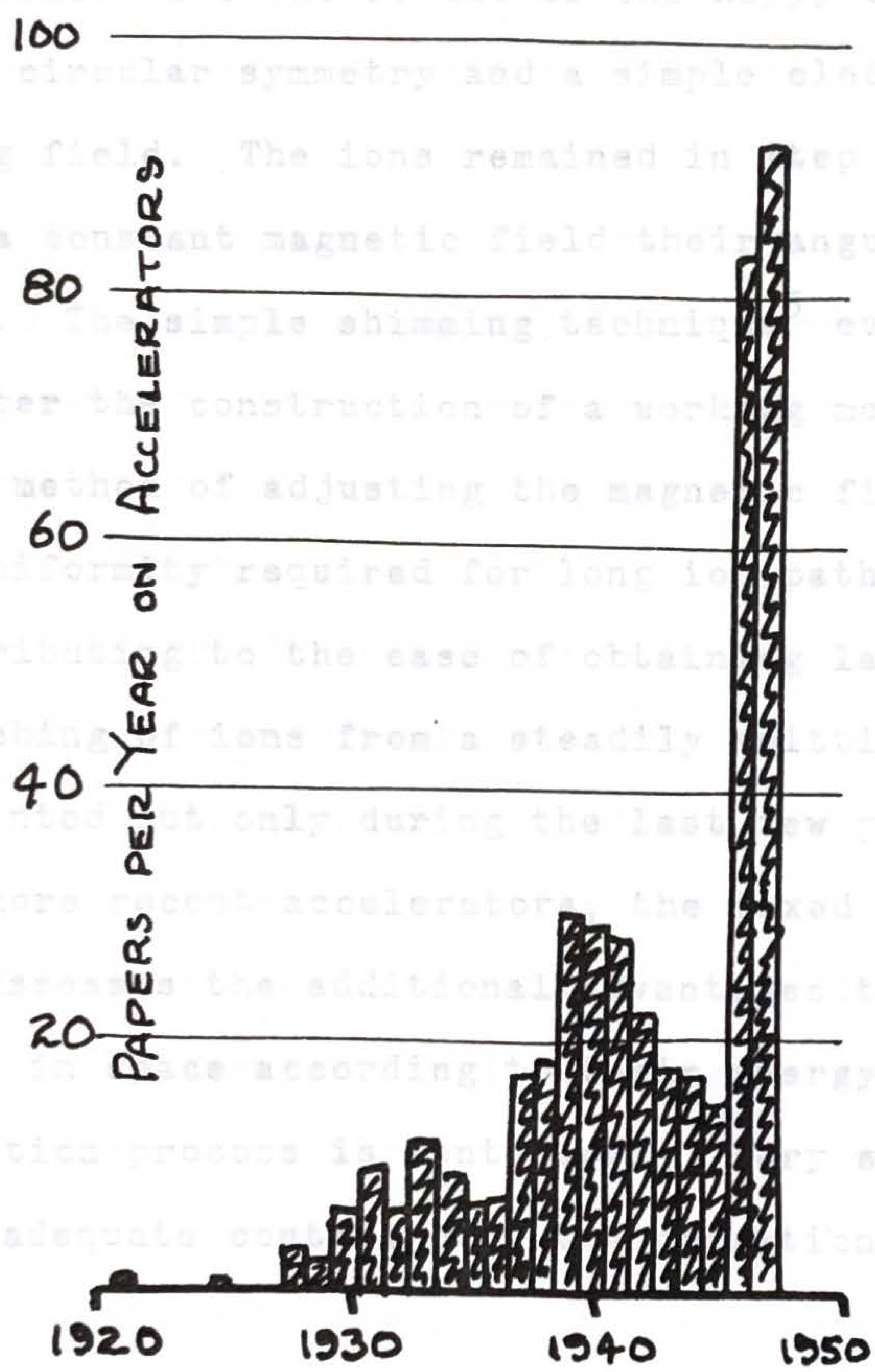


FIG. 1·4

1.5. Lawrence's work with heavy-ion linear accelerators led him to propose the cyclotron[4] in 1930. He expressed doubt whether a measurable quantity of ions would make the complete circuit. The first four inch diameter model, giving 80 K.V. ions, was followed by a second one of nine inches before the exceptional focusing properties of the device were explained.

1.6. These were the result of the happy choice of a magnet with circular symmetry and a simple electrostatic accelerating field. The ions remained in step with the field because in a constant magnetic field their angular velocity is constant. The simple shimming technique[5] evolved by Lawrence after the construction of a working model provided a practical method of adjusting the magnetic field to the degree of uniformity required for long ion paths. Another factor contributing to the ease of obtaining large outputs was the bunching of ions from a steadily emitting source. This was pointed out only during the last few years. Compared with more recent accelerators, the fixed frequency cyclotron possesses the additional advantages that the ions are arranged in space according to their energy and that the acceleration process is continuous. Very simple devices thus permit adequate control over the operation of the machine.

1.7. Because, with the techniques already at hand, the requirements of power and frequency could be met, and because

it required no complicated monitoring systems involving the not-yet-available cathode-ray tube, the cyclotron was the machine of its time. It was an immediate success and has remained one of the most useful of the accelerators.

1.8. In direct contrast to the history of the cyclotron is that of the induction accelerator and its successor, the synchrotron. Many unsuccessful attempts led at last to a very careful evaluation of the conditions for stability and focusing before successful operation of the accelerator was achieved.

1.9. The induction accelerator was first proposed in 1922 by Slepian in an American patent[6] for X-ray generation. He foresaw the toroidal vacuum envelope and the efficient magnetic circuit with D.C. magnetic bias, but neither he nor his assignees (Westinghouse) attempted to build one. In 1928 Wideroe[7] built an induction accelerator for 10 Mev. using a 200 K.V.A. transformer which was switched across the D.C. mains. He realized the central flux condition and provided some focusing but could not launch the beam. Very different apparatus,[8] in which the magnetic field was provided by a solenoid and the acceleration cycle was very short, was tried in Rutherford's laboratory about the same time. Unable to satisfy the central flux condition, Walton here attempted to attain stability with a time-varying radial electrostatic field. It is probably fair to say that the failure of this and all other attempts over the next eleven years was due in about equal measure to two things: the incomplete working

out of the conditions for focusing, stability, and the nature of the possible oscillations; the inadequacy of the measuring gear for establishing control over the motions of the ions.

1.10. One of the early accelerators, the linear accelerator,[9] is now recognised to be an important device, although not till nearly twenty years after its invention were suitable power supplies available to operate it. The multiplier-rectifier accelerator, among the 1930 group, had the distinction of producing the first disintegration by artificially accelerated particles,[9] and has been the most useful of the direct accelerators. The electrostatic belt machine is now a serious competitor.

1.11. In the enthusiasm following the successes of the early thirties, the problem of extending the successful accelerators to higher energy particles was felt to be simpler than it actually was. The upper frequency limit of the cyclotron was estimated from the R.F. power available and the amplification factor obtained in small models, without taking into account relativistic effects. An electrostatic generator designed for over 10 Mev. was built at Round Hill[10] and provided with internal work rooms without regard for the radiation hazard. The construction of the 184 inch magnet at Berkeley was the last gesture of defiance, since three years earlier the limitations on energy available from the cyclotron had been discussed and it was known that an unreasonably large dee voltage would be required.

1.12. The necessity for new methods of acceleration was now evident. During the years of the war, there was a great improvement in radiofrequency power sources, the principle of phase stability was announced by Veksler[11] and McMillan[12] and the eventual success of the induction accelerator paved the way for the synchrotron.

CHAPTER II

GENERAL PROPERTIES OF ACCELERATORS

2.1. Due in part to economic limitations on physical size and in part to the magnitudes of the thresholds for various discharge mechanisms, difficulties of direct acceleration mount rapidly at voltages over a million. When much higher voltages are required, it is necessary to use accelerating devices in which the total voltage representing the energy received by the particle does not appear in the apparatus.

THE INDUCTION ACCELERATOR

2.2. If the charged particle is allowed to circulate in a rising magnetic field, acceleration is accomplished without the presence of a direct voltage. The effect on the particle of one circuit is the same as acceleration by the voltage E which would have been induced in one turn of wire:

$$E = \frac{1}{c}\frac{d\phi}{dt} \qquad (2.2.1)$$

where $\frac{d\phi}{dt}$ is the rate of change of the enclosed flux, and c is the velocity of light. The force F accelerating the particle (mass m, velocity v, charge e) in this circuit is

$$F = \frac{e}{2\pi rc}\frac{d\phi}{dt} = \frac{d(mv)}{dt} \qquad (2.2.2)$$

which shows that the change in momentum of the particle may be found as a function of the change in flux after the particle started. If $(mv)_0$ and ϕ_0 represent momentum and

flux at $t = 0$,

$$mv - (mv)_0 = \frac{e}{2\pi rc}(\phi - \phi_0) . \qquad (2.2.3)$$

The field strength H at the orbit is related to the particle momentum by

$$mv = \frac{Her}{c} . \qquad (2.2.4)$$

A combination of equations (2.2.3) and (2.2.4) gives the relation between the change of flux inside the orbit and the field strength at the orbit:

$$(\phi - \phi_0) = 2\pi r^2(H - H_0) . \qquad (2.2.5)$$

Because the change of flux inside the orbit is a linear function of the field strength at the orbit, a simple magnetic structure may be used to constrain and accelerate the particles. This is the central fact concerning the practicability of an induction accelerator.

2.3. There are two critical field distributions: one in which the field varies as r^0, in which there is no central force on ions above the central plane, hence no vertical stability; another in which the field varies as r^{-1} in which there is no restoring force for a radial displacement, hence no radial stability. For a field distribution in which the field varies as r^n, $-1 < n < 0$, there is both vertical and radial stability. A value of n is chosen which minimizes the coupling between horizontal and vertical oscillations and gives a magnet opening of the most economical shape.

OTHER CYCLIC ACCELERATORS

2.4. A pure electrostatic field is incapable of giving cumulative accelerations to a particle of constant charge.

Dempster[13] showed that the charge on a particle could be varied to obtain several cumulative accelerations in the same electrostatic field. The yields in such a system are, however, very low. Accelerators in which a particle obtains, in an electric field, an energy (in electron volts) greater than any voltage present in the system, depend upon interaction with an electromagnetic wave. In some electron accelerators, the interaction between a simple travelling wave and an electron going at the same speed produces the acceleration. In others, such as the cyclotron, the wave is a fiction of a sort comparable to the rotating field in a single phase induction motor.

2.5. Accelerators differ in the degree to which the particles are focused into the allowable path and in the means by which the particle motion is adjusted to that of the travelling wave. Any accelerator with a long effective path must have efficient focusing. This requirement applies to all heavy particle accelerators. Some electron accelerators, where the mass increases rapidly, have an effective length which is a small fraction of their physical length, and focusing is not required over the whole path. A tendency to return to the correct phase is also required in an accelerator where very many accelerations occur, or where no means are available for adjusting the speed of particle or travelling wave independently.

THE CYCLOTRON

2.6. In the cyclotron, charged particles are released in a constant, uniform magnetic field. The angular velocity ω of a particle in a magnetic field of field strength H,

$$\omega = \frac{He}{mc}, \tag{2.6.1}$$

is independent of energy as long as the mass of the particle is constant. A periodic electric force having the same angular velocity may be arranged to accelerate the particle. Because of the necessity for constant mass, the cyclotron is limited to heavy particles. In practice, the electric force in cyclotrons is produced along a diameter. Because of the curvature of the electric field so produced, an ion moving on a non-central plane will be accelerated toward the centre in the first half of the accelerating gap, and away from the centre in the second half. The net effect is a small displacement toward the centre and the acquisition of a small net velocity which causes the ion to oscillate about the central plane. Toward the periphery, a magnetic focusing also occurs because the bowing of the magnetic lines produces a central force on the non-central particle. These two focusing actions were demonstrated by Lawrence[14] in 1932.

2.7. For the fixed frequency cyclotron, a high uniformity and constancy of magnetic field is required. The constancy is obtained with a current stabilizer, and the uniformity is produced by shimming. Fortunately it is not necessary to have a precise value of field strength at each

point. The field may be considered sufficiently uniform as long as the magnetic disturbances average out over any circular path concentric with the ion path, in which case phase errors will result in negligible loss of acceleration. For reasonable ratios of final voltage to accelerating voltage, this requirement can easily be met with shims along a single radial line. Although the radiofrequency voltages and powers required are high, with modern ion sources the efficiency (measured as the ratio of oscillator power to internal beam power) may exceed ten per cent. A 5 K.W. beam of 7 Mev. protons could be had from the M. I. T. "production cyclotron" for forty dollars an hour (1946 price). This is evidence of the efficient and trouble-free performance which may be obtained from fixed frequency cyclotrons.

2.8. The relativistic mass increase sets an upper limit to the energies over which the ion motion in a cyclotron may be considered isochronous. The time T for one revolution of an ion,

$$T = \frac{2\pi}{\omega} = \frac{2\pi(mc^2)}{Hec} \qquad (2.8.1)$$

is a linear function of the total energy mc^2 of the particle. Thus when the mass of the particle has increased by one per cent (as, for a proton, at about 10 Mev.) the time in the orbit has increased 1 per cent. Shaping the magnetic field to correct for this was at first suggested, but the resulting defocusing was later shown to cause complete loss of the beam. A more critical examination of the electrostatic focusing

mechanism described by Lawrence showed that it is operative only when the ions cross the gap not earlier than the peak of the cycle.[14] This condition cannot be maintained everywhere in a high-energy cyclotron. Magnetic focusing is thus required over the whole cycle, causing a further variation of orbit time during the acceleration. Since the cumulative phase error due to mass increase is proportional to the number of revolutions made by the particle, constant phase error can be maintained by reducing the amplification by a factor equal to the energy increase desired in the output. The dee voltage then increases with the square of the particle energy and the voltage and power demands soon become greater than can be met.

PHASE STABLE ACCELERATION

2.9. Veksler[11] and McMillan[12] about 1945 disclosed the principle of phase stability. They showed that if the frequency of a large cyclotron were set at the value derived from equation (2.6.1) for $m = m_0$, the rest mass of the particle, the particles would be accelerated outward until the mass increase had reduced the phase to zero. If for any reason particles lost energy at this radius, the phase would alter sufficiently to cause the absorption of power to correct the deficiency. The same correcting mechanism would operate for a small decrease in frequency of accelerating voltage, establishing a small increase in particle energy. Thus particles could be accelerated to any energy by changing the frequency in a suitable manner. The

stability provided by this type of operation allowed the use of many more revolutions, thus lower accelerating voltages. Despite the necessary frequency modulation, the radiofrequency problem was, on the whole, simplified. The particle output currents were, of course, considerably reduced.

2.10. In considering the difference between McMillan phase stability and the type of stability possessed by the synchrotron, it is interesting to compare the two general methods used for obtaining stability in electrical and mechanical systems. In one method, one seeks to arrange for the constancy of some important quantity by setting up a relation between it and certain parameters of the system, such that the rate of change of the quantity with respect to each of the designated parameters is as near zero as possible over a range called the operating range. A magnetic voltage stabilizer employs such a method for obtaining stability. In the second method feedback action is used, that is, the constancy of some important quantity is obtained by feeding the departure of the quantity from its preferred value into a control mechanism, the function of which is to reduce the departure. Examples of the second system are the degenerative voltage stabilizer and the flyball governor.

2.11. One point of difference between the two systems is the following. In the first a change in one of the chosen parameters may produce no change at all or a small residual change of either sign in the function under control. In the

second system there is always a residual change, however small, and it has, in the simpler systems, the same sign as the change which would have taken place in the absence of control. Again, in the first method no correcting action is obtained for a change in a parameter outside the chosen set. The second method responds equally to disturbances produced by varying any parameter. Moreover, it is not always possible to find a physical system conforming to an arbitrarily chosen set of conditions required in the first method. Thus the second method, in which independent consideration need not be given to all the possible causes of variation, is more generally applicable. An undesirable characteristic of the second method which is not shared by the first is the possibility of overshoot and oscillation.

2.12. It may be seen that the cyclotron belongs to the first category, and that the ion paths have been chosen to make the phase, that is the time integral of angular velocity, independent of energy. No correction for changes in other variables (such as oscillator frequency) takes place, since no arrangement has been made for setting up a suitable relationship between phase and these variables.

2.13. McMillan phase stability has the characteristics of the second class, since a variation of phase from the prescribed value sets up a control action which returns the phase toward its preferred value. Any factor which would tend in the absence of phase stability to produce a change in phase now produces, in the steady state, a smaller change in the

same direction. Another characteristic of the second class shown by McMillan phase stability is the tendency toward over-correction and oscillation. McMillan phase stability also resembles that of a ball sitting in a depression in a solid.

2.14. The following model, constructed by the writer, demonstrates the difference between the two types of acceleration, the cyclotron and the McMillan phase stable acceleration. A steel ball moves in circles about the axis of a bowl formed by revolving the curve $z = f(r)$ about the z-axis (Figure 2.14). The radial force is $mg\frac{dz}{dr}$ and is equal to the

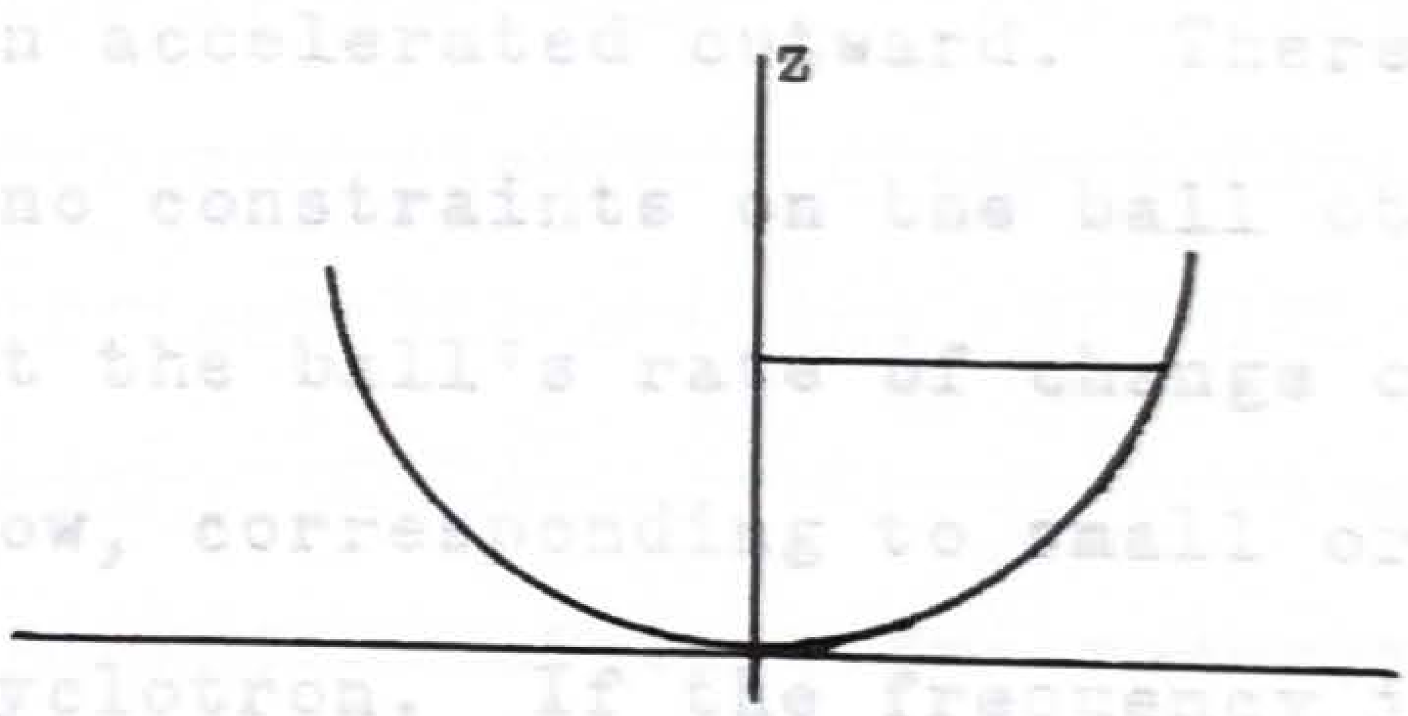


Figure 2.14

acceleration, that is

$$\frac{mv^2}{r} = mg\,\frac{dz}{dr} \qquad (2.14.1)$$

or

$$\frac{dz}{dr} = \frac{v^2}{rg} \quad . \qquad (2.14.2)$$

The angular velocity ω is given by

$$\omega = \frac{v}{r} \quad . \qquad (2.14.3)$$

On the isochronous surface, the angular velocity is constant

$$\omega = k \; . \qquad (2.14.4)$$

Equations (2.14.2), (2.14.3) and (2.14.4) give the condition

$$\frac{dz}{dr} = \frac{k^2 r}{g} .$$

If the vertex of the bowl is assumed to be at $z = r = 0$, this gives

$$z = \frac{k^2 r^2}{2g} . \qquad (2.14.5)$$

Thus the isochronous surface is a paraboloid the equation of which is (2.14.5).

2.15. If a steel ball is placed in such a paraboloidal bowl and a small periodic motion is given to the bowl, the resulting motion of the ball remains small unless the frequency of the motion given to the bowl is the resonant frequency f,

$$f = \frac{\omega}{2\pi} = \frac{k}{2\pi} . \qquad (2.15.1)$$

The ball is then accelerated outward. There are, of course, no grooves and no constraints on the ball other than the surface, so that the ball's rate of change of radius may be made fast or slow, corresponding to small or large amplification in the cyclotron. If the frequency is slightly off resonance, the radius will first increase then decrease. If, when the ball is being accelerated outward, the frequency of the motion given to the bowl is changed, the ball will spiral in again. This type of acceleration thus corresponds closely to the acceleration experienced by a particle in a fixed frequency cyclotron.

2.16. One now accelerates the ball in a nearly paraboloidal bowl constructed according to the equation $z = \frac{k^2 r^2}{2g}$ in which k has a value k_0 at the axis and alters slightly as the radius increases. The difference between k_0 and the value of k at

a radius r is made proportional to r^2. There is now a slow change of ω with the amplitude of the ball's motion. This change is similar to that of the angular velocity of a particle in a uniform magnetic field due to mass increase. Acceleration in the central regions of this bowl is much as it was in the truly paraboloidal bowl, but instead of being accelerated continuously outward as before, the ball reaches a point where it rotates at a fixed radius. The ball at this radius has McMillan phase stability, and if the frequency is slowly changed, if will be accelerated outward correspondingly.

2.17. Although, in theory, the principle of phase stability makes possible acceleration to any value of energy, in practice when high energies are desired the necessary frequency modulation is difficult to obtain. The 184 inch cyclotron at Berkeley, producing 350 Mev. protons, uses an external condenser with an ingenious circuit for reducing end effects. The 500 Mev. Columbia synchrocyclotron uses an internal condenser. Practical considerations prevent the use of a time varying magnetic field, though this would be just as effective in increasing particle energies.

2.18. It seemed likely, in 1947, that a solution to the frequency modulation problem would be found for a 1 Bev. machine. The frequency is required to change in the ratio 2:1 (protons) although to establish a useful range of 2:1 a somewhat larger range is necessary so that the most suitable

part of the frequency-time curve may be chosen. The radius of the pole face is given by

$$R^2 = \frac{T^2 + 2TE_0}{83.49\ H^2}(1.19)^2 \quad , \qquad (2.18.1)$$

H = field strength in Kilogauss
T = kinetic energy in Mev.
R = orbit radius in feet
E_0 = rest mass of particle in Mev.

The factor 1.19 is based on model studies of magnets.

THE PROTON SYNCHROTRON

2.19. The construction of such a machine was contemplated at Brookhaven, but its weight was estimated to be 10,000 tons. The industrial effort represented by the outlay of such a large amount of iron makes it seem worthwhile to consider a more complicated but more economical type of accelerator for outputs of this energy. If a ring of iron is used instead of a solid magnet, the magnetic field may be changed during acceleration to keep the particles confined to a narrow range of radii. While it is not accurate to say that the cost of such a device is a linear function of energy, the ring magnet represents a great saving over a solid magnet for the same energy. The principle of phase stability brings such an arrangement within the bounds of possibility, for otherwise tracking problems would be insuperable.

PHASE OSCILLATIONS

2.20. Acceleration in the phase stable region has the disadvantage that phase oscillations accompanied by radial oscillations may take place. In order to have stability in

a rising magnetic field, the amplitude of the accelerating voltage must be greater than the energy (in volts) to be added to the particle per revolution. If the field strength varies with the radius of the particle orbit, the angular velocity of the particle will also vary with radius, so that for a given angular velocity of the accelerating voltage there will be a value of radius at which the particle and the radio frequency voltage have the same angular velocity. This is the equilibrium orbit. At this radius there is a phase where a particle gains per revolution just the right amount of energy to keep up with the changing equilibrium energy. This will be the stable phase. If a particle on the equilibrium orbit has such a phase that more energy than required is gained per revolution, it will increase its radius and start to move in phase toward the stable phase. Equilibrium will not be reached until the net energy gain is appropriate to the new radius, and at this point the phase will be changing in the direction to decrease the energy gain per revolution. This type of correcting action produces oscillation.

2.21. For a small change in radius, the rate of change of phase with the number of revolutions is proportional to the change in radius. The rate of change of radius with the number of revolutions is proportional to the ratio of excess energy to total energy. Thus the second derivative of phase with respect to number of revolutions is proportional to the ratio of excess to total energy. As was shown by McMillan,[12] this is of the

same form as the equation of motion of a pendulum acting under a constant torque. When this torque is adjusted to make the angle of rest equal to the stable phase angle in the system above, the phase oscillation caused by a phase error corresponds to the angular oscillation of the pendulum displaced by the same angle. The stable phase range also corresponds to the angles which the pendulum may occupy while oscillating at constant amplitude. The oscillations which occur at injection arise from the injected particle arriving with the wrong energy for the orbit and/or a phase other than the stable phase.

2.22. The Berkeley frequency modulation tests on the 37 inch model early in 1946 were a convincing demonstration of the possibilities inherent in the phase stability principle. The eventual operation of the betatron in 1940 had shown that acceleration involving miles of particle path was a practical matter. In 1946, the construction of the first proton synchrotron was begun at Birmingham University. A small model was operated successfully at 6 Mev. in May, 1949, at Berkeley.

2.23. The proton synchrotron thus stands (for the time being) at the end of a series of accelerators in which fundamental simplicity has been sacrificed progressively to raise output energy and to lower cost. Successful operation will depend upon being able to follow the complicated motions of the particles over the whole acceleration cycle, and upon solving the problems of injection and extraction.

CHAPTER III

INJECTION

METHODS OF INJECTION

3.1. Many methods of injecting protons into a proton synchrotron have been considered by the three groups engaged at present in the construction of the synchrotron. The method originally proposed for use at Birmingham was injection of the accelerated ions into the orbital plane from above, followed by downward displacement by a vertical electric field. The Berkeley group at one time considered injection at the inner limit of the chamber followed by cyclotron acceleration with fixed field and fixed radio frequency. A large separation of successive revolutions can be obtained with a moderate value of accelerating voltage. Another scheme, proposed at Brookhaven, involved using the injected particles which lose energy by crossing the gap when the accelerating voltage is applied in the reverse direction. When the chamber is filled with particles, a 180 degree phase shift is applied in a short time to the radio frequency. Most of the particles are now in the stable range and are trapped.

3.2. The only proton synchrotron which has yet been operated is the one at Berkeley, which was built on a small scale to test the theory. Protons accelerated in a small cyclotron are injected at the outside of one of the straight sections in the accelerating magnet. The mechanical system

allows accurate alignment of the beam in both directions. A deflector electrode bends the beam until it is parallel to the ion paths. It has been found that the vertical and radial oscillations induced by the random irregularities of the field play an important role in getting a large fraction of the beam clear of the injector electrode. An artificial obstruction corresponding to the thickening of the tip of the electrode from 1/32 inch (a practical value for construction) to 13/32 inch was found to produce no appreciable change in the number of ions accepted. The experience with the California model synchroton has shown that horizontal injection in the median plane from the outside is a satisfactory solution to the problem of getting protons into the stable orbits, and the Brookhaven and Birmingham groups now favour such a system.

CONDITIONS FOR AVOIDING BETATRON OSCILLATIONS

3.3. It is evident that to inject the greatest number of particles, the energy of the particles injected must be continuously adjusted to that of the stable orbit nearest the injector. If particles should be injected at fixed energy, the betatron oscillations, due to the particles being displaced from their orbits, would fill the chamber when the first particles had reached the equilibrium orbit, thus making further injection useless. Variable energy injection may be continued until the first particles have crossed the chamber; thus twice the injection time is available. Fixed energy injection also involves a loss of particles due to

the betatron oscillations being added to the radial oscillations due to phase oscillations. About three times as many particles may be injected when variable energy injection is used.

3.4. The advantages of variable energy injection have been recognized from the beginning in considering possible methods of injection. The difficulties of producing the necessary variation of energy are, however, great. For the Birmingham proton synchrotron where the field at injection rises at the rate of 20,000 gauss$\cdot$sec^{-1}, the rate of rise required is 90 megavolts per second when injection occurs at 220 gauss and injection lasts 220 microseconds. In the model synchrotron operated at Berkeley, the ion source was a cyclotron, and variable energy injection could not be considered. The large synchrotron now being built at Berkeley will use ions at 10 Mev. from an Alvarez linear accelerator. Thus variable energy injection would be very difficult in this case. Variable energy injection of the type to be discussed could, however, be applied to the Van de Graaff source which is to be used at Brookhaven. As far as we know, neither the Brookhaven nor the Berkeley group plans to use variable energy injection at present.

3.5. The writer's concern with the injection system was the problem of finding a practical method of realizing the well known advantages of avoiding betatron oscillations. Such a method was arrived at as a result of his investigations and is described below. The energy of the stable orbit at the injector is, for 20,000 gauss$\cdot$sec^{-1} rate of field strength rise and injection at 220 gauss,

$$E = 0.5 + 90\,t \qquad (3.5.1)$$

E = energy in Mev.
t = time in seconds .

This relation between energy and time is obtained by accelerating protons with a voltage in which a time-varying voltage is added to a fixed voltage of 500 K.V. A departure of 1/10 per cent from the correct value would result in oscillations having an amplitude of 0.2 cm. The highest percentage accuracy was required in the 500 K.V. fixed voltage. A time-varying guiding voltage of the same order of accuracy is also required and, for this, suitable apparatus was designed by the writer.

3.6. In the remainder of this chapter the methods evolved for: (1) producing a rising voltage and adding it to a 500 K.V. source and (2) stabilizing a 500 K.V. source to an accuracy of 1/10 per cent, will be discussed.

PRODUCTION OF RISING VOLTAGE

3.7. When the rate of rise required is 90 M.V.$\cdot$sec^{-1}, the ion source current of 1 milliampere is small compared with the current required to change the high voltage parts of the injection apparatus. These parts have a capacity to ground of about 140 micro-micro-farads. The charging current will be

$$\frac{dQ}{dt} = C\,\frac{dV}{dt}\,, \quad \frac{dV}{dt} = 90 \times 10^6 \text{ volt}\cdot\text{sec}^{-1};$$

that is

$$\frac{dQ}{dt} = 12.6 \text{ milliamperes}.$$

This current must be supplied for 220 microseconds. It is possible to use a vacuum tube circuit with a sufficient number of tubes in series to withstand 30 K.V. The design of such a

circuit is straightforward, since no plate dissipation need occur except during the pulse. The use of an output transformer offers considerable simplification in overall insulation and power supplies. The design of such a transformer will now be considered.

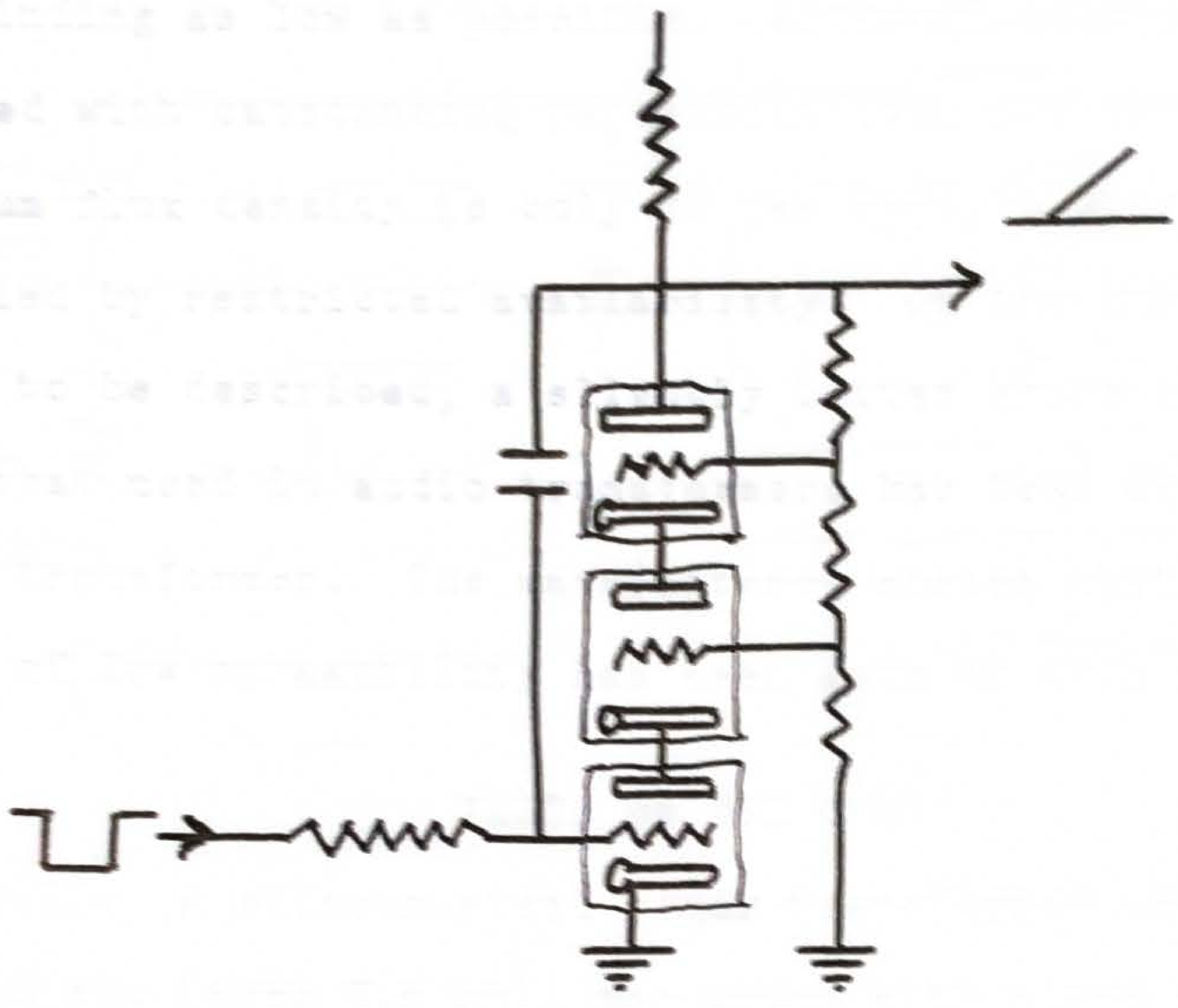


FIG 3.7

DESIGN OF THE TRANSFORMER

3.8. For a 220 microsecond pulse and a repetition frequency of once in ten seconds, the duty cycle is one in $5x10^4$. Thus the heating of the core is negligible and the oil will remain cool, materially assisting avoidance of breakdown. The heat produced per centimeter in 220 microseconds with a current of 10 amperes in No. 40 wire is $4.1x10^{-5}$ gram calories. The corresponding temperature rise is $4^{\circ}C$ so that even finer wire

could be used. Thus the volume of the copper in the windings is negligible. The resistance of the primary winding has, however, an upper limit, as will be shown later.

3.9. The highest maximum flux density is required of the core in order to keep the capacity and leakage inductance of the winding as low as possible. Although alloys have been developed with outstanding permeabilities, the improvement in maximum flux density is only 30 per cent, a slight advantage annulled by restricted availability. On the basis of tests about to be described, a slightly better grade of silicon steel than that used in audio transformers has been chosen for the final transformer. The manufacturer states that the surface layer of low permeability has been made as thin as possible.

TESTS ON THE CORE

3.10. A silicon steel power transformer core of section 4 x 2.4 cm. (area 9.6 cm^2.) was wound with a low resistance winding of 400 turns. A source of current of high capacity was provided: six 807 tubes in parallel, with a plate supply of 2000 volts. The peak currents obtainable were of the order of 2 amperes, when the screen voltage was slightly in excess of 300 volts. The voltage developed across the winding is:

$$E = N\frac{d\phi}{dt} \times 10^{-8} \qquad (3.10.1)$$

N = number of turns
ϕ = enclosed flux
E = voltage across the winding .

Thus,

$$d\phi = \frac{E}{N} \times 10^{+8}\ dt$$

and

$$\phi_{max} = \frac{10^{+8}}{N}\left[\int E\ dt\right]_{max} . \qquad (3.10.2)$$

The coil was connected in a Miller circuit with the tubes supplying the current and a positive square wave of large amplitude (1000 volts) applied to the grids which were normally biased to cut off. When the voltage applied to the lower end of the

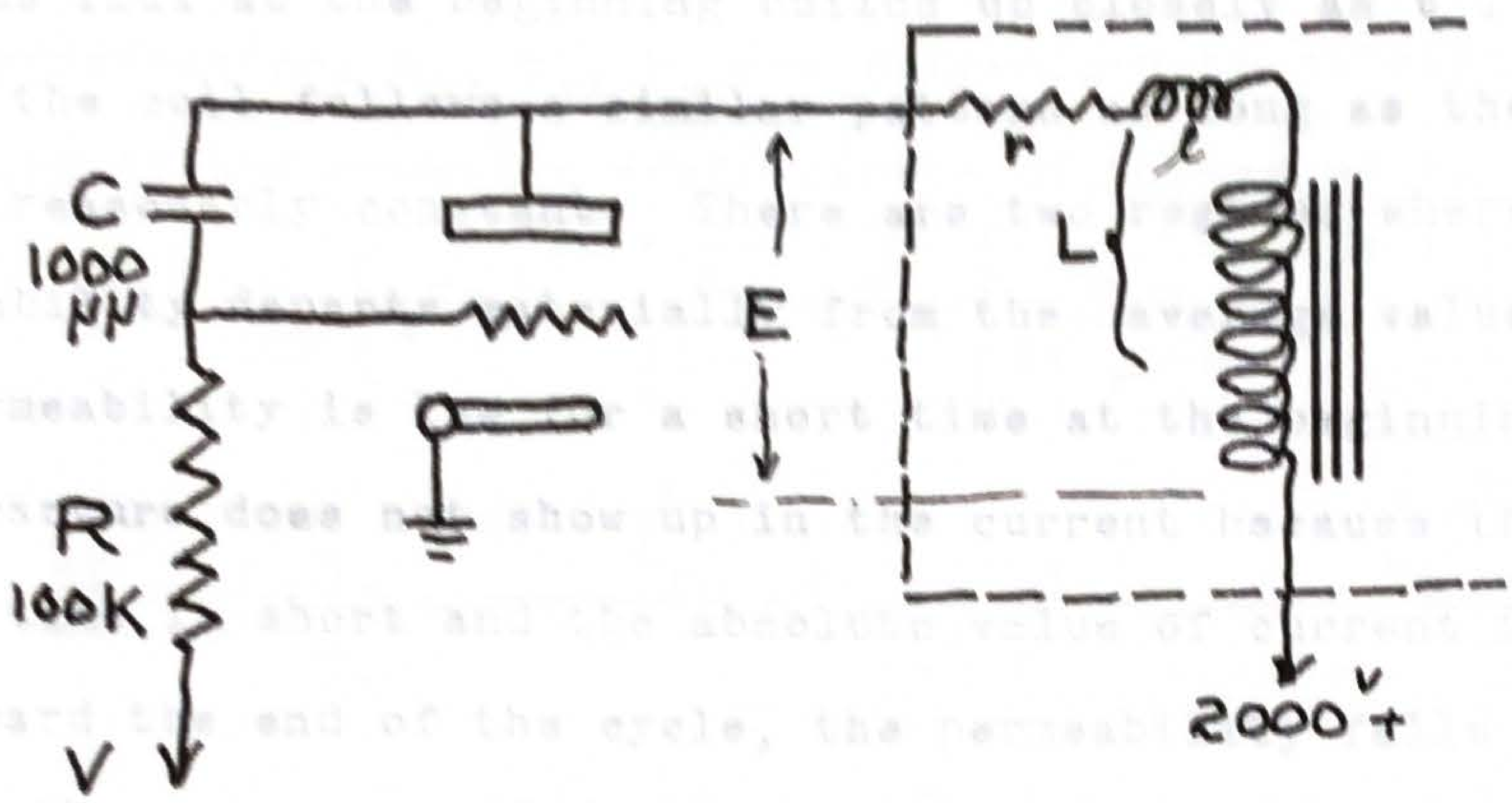


FIG 3.10

resistor becomes positive, current starts to flow in the core under the control of Miller feedback. The range of grid swing is about 25 volts, so that with an applied voltage of 1000, the rate of change of the voltage at the plate need only decrease 2.5 per cent to result in full current.

3.11. The flux starts to build up in such a way that the current I in the condenser is given by

$$I = C\,\frac{dE}{dt} \cong \frac{V}{R} = 10 \text{ milliamperes} \qquad (3.11.1)$$

neglecting r and ℓ in figure 3.10 ($r = \ell = 0$). Substituting

from (3.10.1) this gives

$$\frac{d^2\phi}{dt^2} \cong \frac{V \times 10^8}{NCR} \qquad (3.11.2)$$

so that when ϕ and $\frac{d\phi}{dt}$ are zero for $t = 0$,

$$\phi \cong \frac{V \times 10^8}{2NCR} t^2 , \qquad (3.11.3)$$

Thus flux at the beginning builds up closely as t^2. The current in the coil follows a similar pattern as long as the permeability is reasonably constant. There are two regions where the permeability departs materially from the average value. The permeability is low for a short time at the beginning, but this departure does not show up in the current because the length of time is short and the absolute value of current is small. Toward the end of the cycle, the permeability falls rapidly and the current and its departure from the ideal relation are both large. This has a bearing on the design of the transformer, since it sets a limit on the resistance of the winding.

3.12. An estimate of the circuit errors from the beginning of the cycle up to the beginning of the final drop in permeability may be made by obtaining a closer approximation to the solution of the circuit of figure 3.10. The inductance of the core is regarded as constant and the E_g versus I_p relation is taken as

$$I_p = -G_m E_g . \qquad (3.12.1)$$

The other circuit equations are

$$E_p = I_p r + \frac{dI_p}{dt} L \qquad (3.12.2)$$

$$C\frac{d}{dt}\left(I_p r + \frac{dI_p}{dt}L + \frac{I_p}{G_m}\right) = \frac{V - \frac{I_p}{G_m}}{R} \qquad (3.12.3)$$

whence we obtain the second order linear differential equation

$$L\frac{d^2I}{dt^2} + (r + \frac{1}{G_m})\frac{dI}{dt} + \frac{I}{G_m RC} = \frac{V}{RC} . \qquad (3.12.4)$$

The solution has been shown to be, (3.11.3) essentially

$$I = k_o t^2 .$$

If $I = k_o t^2 + k_1 t^3 + k_2 t^4$ is taken as the form of the next approximation, k_o, k_1, k_2 are determined by substitution in (3.12.4) and equating coefficients of powers of t. Thus

$$I = \frac{V}{2RCL}\left\{t^2 - \frac{(r + \frac{1}{G_m})}{3L}t^3 - \left[\frac{1}{12G_m RCL} - \frac{(r + \frac{1}{G_m})^2}{12L^2}\right]t^4\right\} \qquad (3.12.5)$$

$$\frac{dI}{dt} = \frac{V}{RCL}\left\{t - \frac{(r + \frac{1}{G_m})}{2L}t^2 - \left[\frac{1}{6G_m RCL} - \frac{(r + \frac{1}{G_m})^2}{6L^2}\right]t^3\right\} \qquad (3.12.6)$$

$$\frac{d^2I}{dt^2} = \frac{V}{RCL}\left\{1 - \frac{(r + \frac{1}{G_m})}{L}t - \left[\frac{1}{2G_m RCL} - \frac{(r + \frac{1}{G_m})^2}{2L^2}\right]t^2\right\}. \qquad (3.12.7)$$

Equation (3.12.6) gives the error in terms of departure of induced voltage from a linear function of time, and equation (3.12.7) gives the error in terms of the departure from a constant value of the slope of the curve in (3.12.6). Representative circuit values for the core test were

$$\begin{aligned} R &= 100{,}000 \text{ ohms} \\ C &= 1 \times 10^{-9} \text{ farads} \\ L &= 0.242 \text{ henries} \\ r &= 10 \text{ ohms} \\ V &= 1000 \text{ volts} \\ 1/G_m &= 25 \text{ ohms} \\ G_m &= 0.04 \text{ amperes per volt} \\ t_{max} &= 220 \times 10^{-6} \text{ seconds} \end{aligned} \qquad (3.12.8)$$

It will be seen from (3.12.6) and (3.12.7) that the first error term, the coefficient of t^2 in (3.12.6) and the coefficient of t in (3.12.7), contains the winding resistance in series with a term $\frac{1}{G_m}$ representing the driving impedance. The departure of the voltage from its correct value at the end of the sweep due to the first error term is obtained from (3.12.6) by substituting the numerical values (3.12.8) and is 1.6 per cent. Similarly the slope error is 3.2 per cent. The design of the transformer should ensure that the transformer primary resistance is smaller than the circuit term. It should be noted that figure 3.10 has been simplified by the omission of the grid to ground capacity and that a more exact expression for the driving impedance is obtained in section3.22. The second error term, the coefficient of t^3 in (3.12.6) and the coefficient of t^2 in (3.12.7), is the difference of two quantities the first of which is dominant and results from the voltage required to drive the grid. This effect contributes to the second error term rather than the first because the current flow varies as t^2. The voltage error at the end of the cycle due to grid drive is 0.8 per cent. The slope error is 2.5 per cent.

3.13. As long as the primary inductance was assumed constant, there was no need to consider primary leakage reactance separately since it had no effect on the shape of the relation between flux and time. When the core begins to saturate, an increasing fraction of the total voltage is

developed across the leakage reactance. This makes the induced voltage at the end of the cycle lower than it should be. The primary current increases over its constant-inductance value, and causes a corresponding increase in both the resistance error and the grid drive error described in section 3.12. Higher order terms are thus introduced, which are difficult to compensate for. The maximum flux density has been reduced because of this consideration. The first two error terms can easily be dealt with by positive feed back. A special winding has been provided for this purpose.

3.14. The final part of the cycle, in which the core approaches and reaches saturation, is difficult to describe mathematically. A qualitative description of the behaviour of the circuit in this region will now be given. From the complete oscillogram, the maximum flux density of the core may be calculated. When a sufficient number of turns has been provided to more than saturate the core with the current available, the oscillogram of voltage is as shown in figure 3.14.1. When the permeability of the core begins to drop

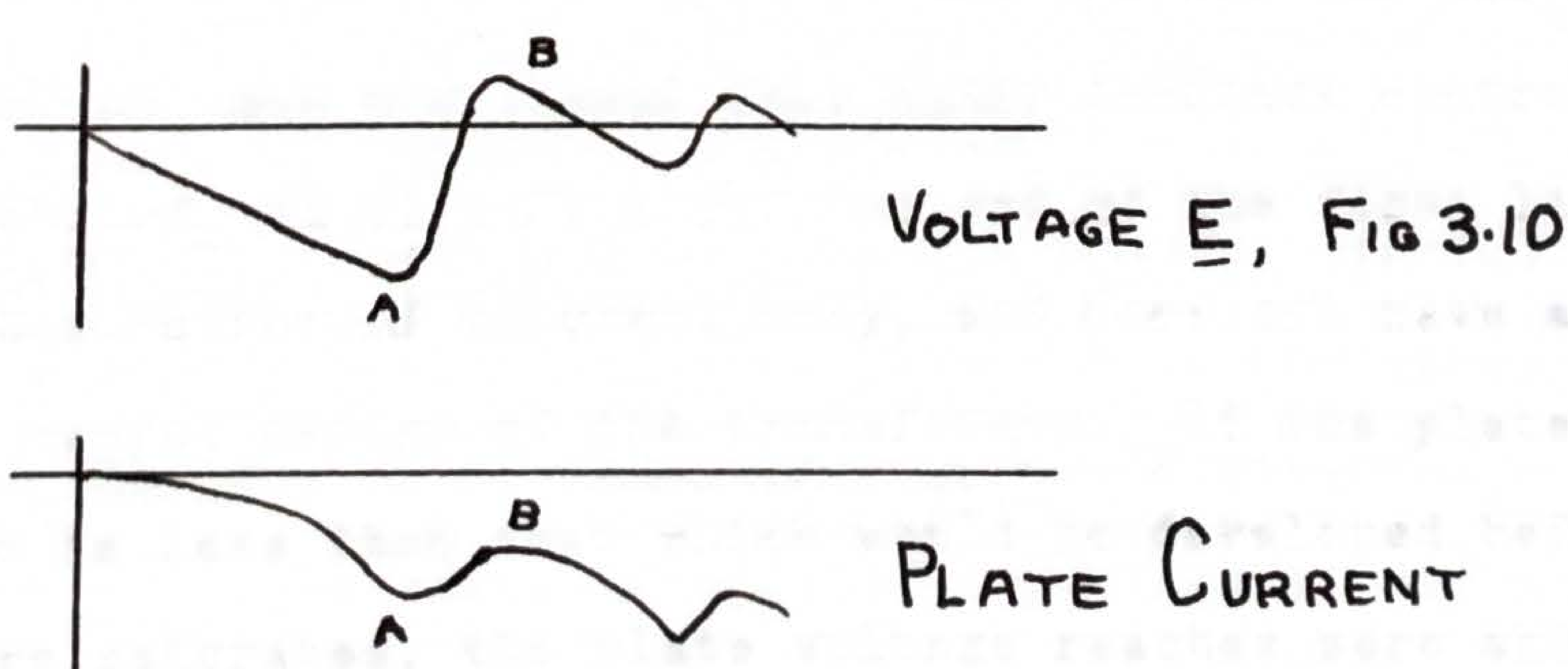


FIG 3.14

seriously, the grid voltage goes more and more rapidly toward zero. Because of the large amount of feedback, the departure of ϕ from the value given in (3.11.3) does not become marked until the grid is near zero, then increases very rapidly. As soon as the absolute value of the voltage E, figure 3.10, stops increasing (A), control by the feedback circuit is lost, and the feedback is inoperative. As long as the plate voltage is moving in the positive direction, the grid is effectively grounded, and the circuit executes an oscillation in which the feedback condenser is charged with the plate end positive. The oscillation carries the plate potential above that of the supply (B), and the plate current below its maximum value. At the voltage extreme (B), the direction of current in the feedback condenser reverses, and the feedback circuit again takes over, causing the absolute value of the voltage to increase at a rate identical with that of the initial rise. When the whole transformer is added, the main capacity in the oscillating circuit is that reflected from the secondary into the primary. One can then change independently the rate of the voltage rise from A to B, that is the 'period' of the half oscillation, and the linear fall under feedback control. The behaviour of the circuit after the end of the first linear rise is a matter of interest only, and does not have any part in the useful design of the transformer. If the plate supply voltage is less than that which would be developed before the core saturates, the plate voltage reaches zero and the oscillogram then is shown in figure 3.14.2. The appearance

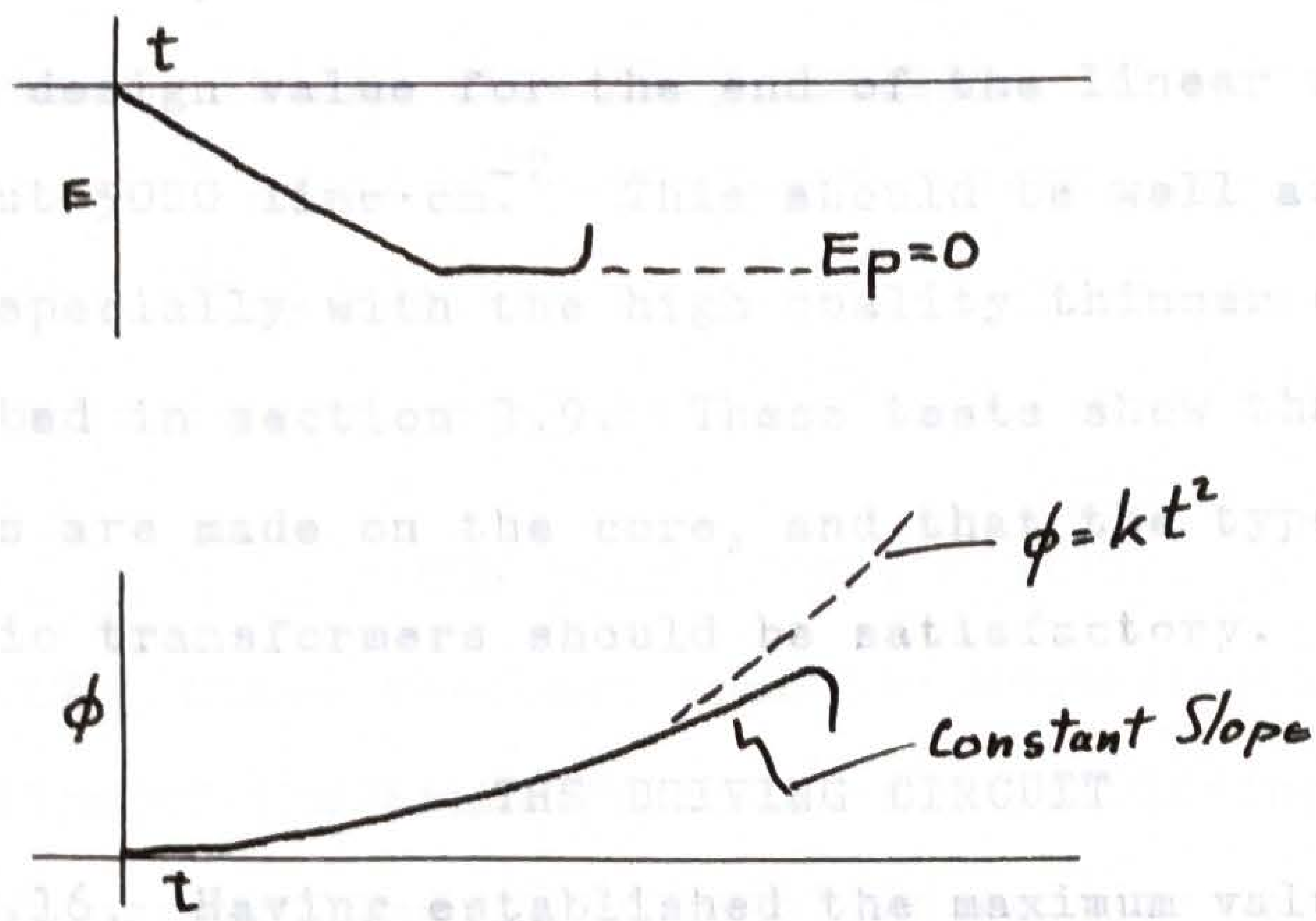


FIG. 3.14.2

of a flat part indicates that the core is being run conservatively. In this case, the grid loses control when the plate voltage reaches zero, and during the flat part of the voltage curve, the current is controlled by the inductance.

3.15. As long as ample ampere turns are provided, the increase in voltage occurs at saturation, and the relation (3.10.2) may be used to calculate the flux density in the core, the value of $\left[\int E dt\right]_{max}$ being obtained from the oscillogram of the voltage developed across the winding. The value obtained for the silicon steel power transformer core was about 10,000 line.cm.$^{-2}$ Allowing for a space factor of 0.9, this is still low for silicon steel. The low value is probably due to the shielding effect of eddy currents in the core, since the eddy current shunt resistance in the equivalent circuit (section 3.21) is rather low, that is, close to the working

values of impedance for the winding to which it is referred. A safe design value for the end of the linear rise seems to be about 5000 line·cm^{-2}. This should be well away from saturation especially with the high quality thinner core material described in section 3.9. These tests show that no great demands are made on the core, and that the type of core used in audio transformers should be satisfactory.

THE DRIVING CIRCUIT

3.16. Having established the maximum value for flux density in the core, we should now arrange the circuit and the rest of the transformer to obtain a rise as nearly linear as possible in the voltage across the load. It would be desirable to have the rate of change feedback derived from the top of the acceleration column, so that variation of the rate of change caused by intermediate elements could be corrected. Failing this, one might hope to derive the feedback from the secondary, at least. One reason that this is not possible is clear from the values given in the equivalent circuit of section 3.25. In addition, derivative feedback circuits are more difficult to close because of the 6 db. per octave rise which extends to the end of the useful region. It is possible that satisfactory feedback from the secondary could be obtained by defining closely in the feedback loop the frequency above which feedback is not required. However, it will be shown that it is possible to obtain an equivalent circuit between

the tube and the final load, which is nearly aperiodic and which approaches a linear rise very quickly. The driving circuit to be used is a one stage Miller circuit similar to that described for core tests. The derivative feedback starts to fall off at a frequency near the principal resonance of the loaded transformer.

THE TRANSFORMER WINDINGS

3.17. Since feedback from the secondary cannot be used, the design of the transformer secondary winding must ensure that the flux versus time curve provided results in a linear rise of the voltage in the secondary. The non-ideal elements occuring in the secondary circuit are the leakage inductance, the winding resistance, the distributed capacity and the capacitive load.

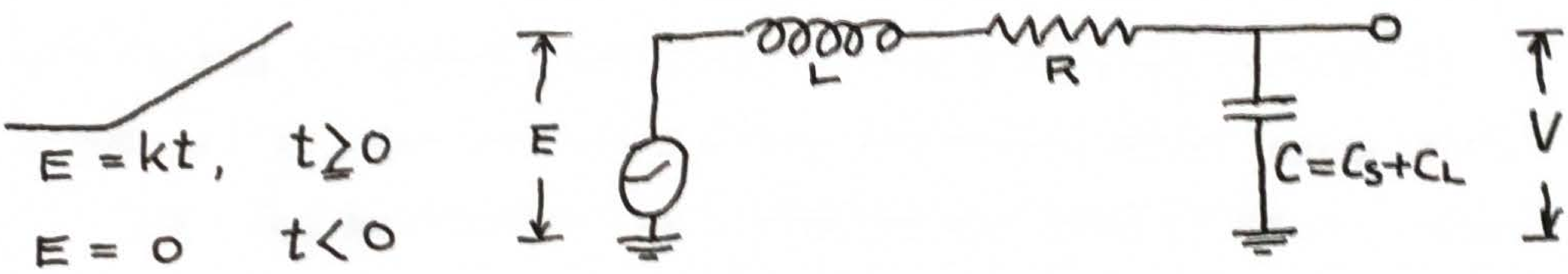


FIG. 3.17.1

The current in this circuit is found from the second order linear differential equation

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C} I = k . \qquad (3.17.1)$$

If $\frac{4L}{C} > R^2$, the solution shows the usual damped oscillations. Using the initial conditions $I = 0 = \frac{dI}{dt}$ when $t = 0$, we obtain in this case

$$I = Ck - \frac{2k\sqrt{LC}}{\sqrt{\frac{4L}{C} - R^2}}\, e^{-\frac{Rt}{2L}} \sin\left(\frac{\sqrt{\frac{4L}{C} - R^2}}{2L} t + \beta\right) \qquad (3.17.2)$$

where

$$\beta = \tan^{-1}\frac{\sqrt{\frac{4L}{C} - R^2}}{R} . \qquad (3.17.3)$$

The voltage V at A, evaluated from the relation

$$V = \frac{1}{C}\int_0^t I\,dt \qquad (3.17.4)$$

is

$$V = kt + \frac{2kL}{\sqrt{\frac{4L}{C} - R^2}}\, e^{-\frac{R}{2L}t} \sin\left(\frac{\sqrt{\frac{4L}{C} - R^2}}{2L}t + 2\beta\right) - kRC . \qquad (3.17.5)$$

The voltage output exhibits damped oscillations about the line

$$V = kt - kRC . \qquad (3.17.6)$$

The frequency of the oscillations is

$$\frac{\sqrt{\frac{4L}{C} - R^2}}{4\pi L} \left(\cong \frac{1}{2\pi\sqrt{LC}} , \quad R << \sqrt{\frac{4L}{C}} \right) \qquad (3.17.7)$$

and the amplitude of the oscillations dies out as $e^{-\frac{Rt}{2L}}$. Thus the voltage at A, figure 3.17.1, eventually rises at the same rate as the voltage E, but along a line displaced vertically by RkC volts. The non-oscillating solutions, which occur when $\sqrt{\frac{4L}{C}} \leqq R$, can be obtained in a similar way from (3.17.1). They show the voltage approaching the same line, (3.17.6), but without oscillations. It is important that a steady rise of voltage should be reached as quickly as possible. For a pure resistance and capacity, the time constant should be small compared to 220 microseconds. For a circuit with resistance, inductance and capacity, when the inductance has first been made as small as possible, the circuit should be damped somewhat under critical damping. As with the case of a galvanometer, the criterion is the time taken to approach to within

a certain distance of the final line. In our case this is about 500 volts from it.

3.18. The equivalent circuit of a transformer is given in figure 3.18.1.

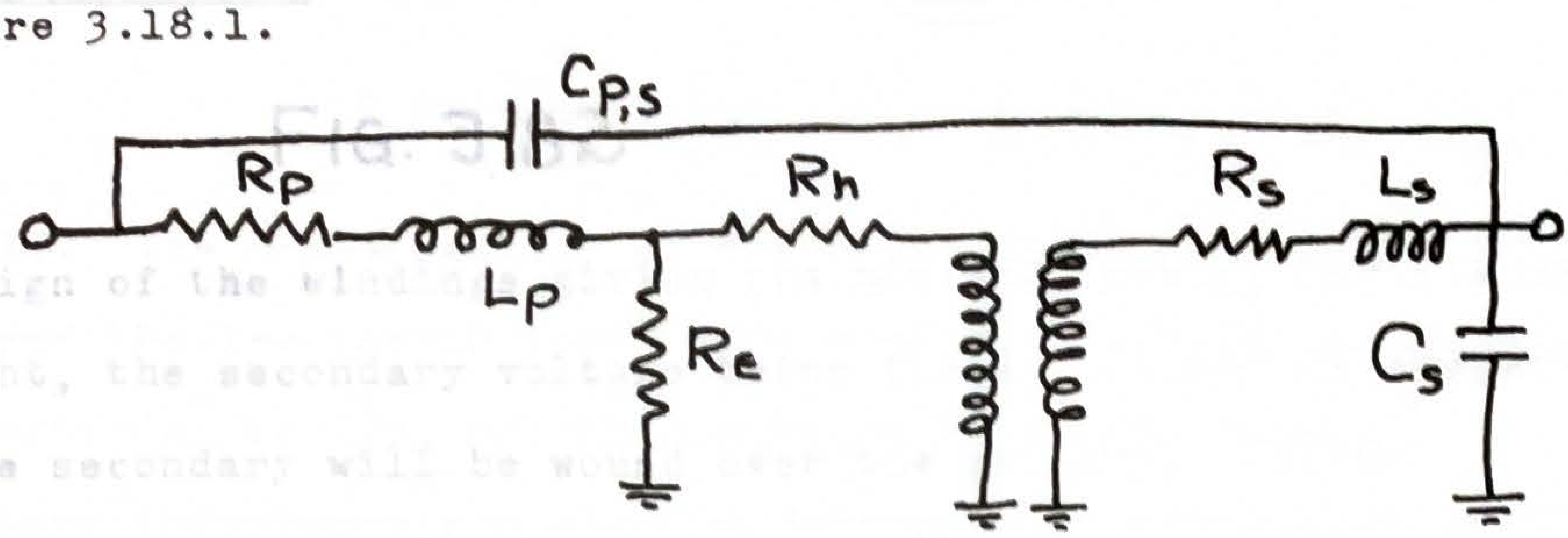


FIG. 3.18.1

R_p resistance of primary winding

R_s resistance of secondary winding

L_p leakage inductance of primary

L_s leakage inductance of secondary

R_e shunt resistance due to eddy currents

R_h resistance due to hysteresis loss. Because each part of the cycle is considered separately, this element in the equivalent circuit plays no useful role.

C_p, C_{ps}, C_s stray capacities, primary to ground, primary to secondary, and secondary to ground.

The leakage inductance L of a transformer referred to one winding is

$$L = \frac{4\pi N^2 \; 2\pi r \; t}{10^9 \ell} \qquad (3.18.1)$$

N = number of turns
r = radius of the winding
t = thickness of the winding
ℓ = length of the winding

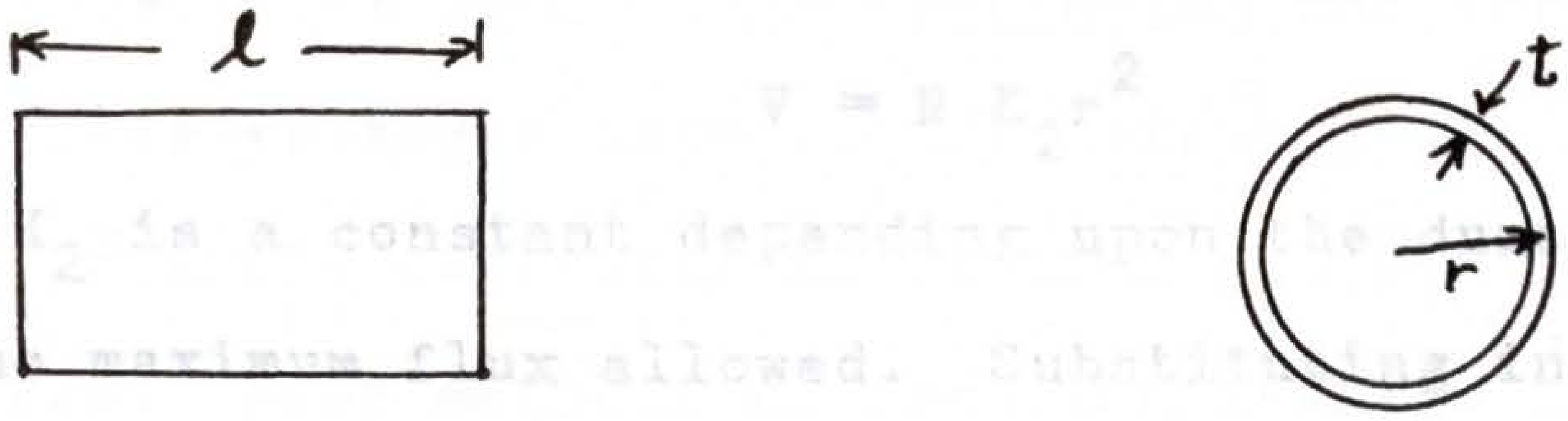


FIG. 3.18.2

The design of the windings giving the minimum leakage inductance is sought, the secondary voltage being fixed. It may be assumed that the secondary will be wound over the primary. Inter-leaving is not attractive because the voltage is so high. The finest convenient size of wire will be used, and the wire will occupy negligible space. Care is, of course, required to prevent the production of high gradients by making sure that a single strand of wire is never isolated. The high voltage lead is brought away from the transformer in tubular weave. Whether a layer is started at the same end as that at which the lower layer left off, or the opposite one affects the distributed capacity, but not the thickness of dielectric required, unless special methods are provided for isolating the lead crossing over. It was decided to start the new layer at the same end as the previous layer finished, since this procedure gives the simpler winding. For the type of winding being considered, then, the thickness is constant, depending only upon the final voltage required:

$$t = 2 \times \frac{V}{K_1} \qquad (3.18.2)$$

where K_1 is the working stress of the dielectric in volt.cm.$^{-2}$.

The voltage output is

$$V = N\,K_2 r^2 \qquad (3.18.3)$$

where K_2 is a constant depending upon the duration of the pulse and the maximum flux allowed. Substituting in (3.18.1),

$$L = \frac{8\pi^2}{10^9 K_2^2} \cdot \frac{V^2 t}{r^3 \ell} \quad . \qquad (3.18.4)$$

3.19. The cross section of the core should therefore be large, and the leg length long. The resistance of the winding is proportional to V/r, and this must be kept within bounds. The leakage inductance can also be decreased by using a material of high dielectric strength, leading to a small value of t. Polytetrafluorethylene sheet has proved to have excellent properties, electrically and mechanically and will withstand transformer oil indefinitely. The sheet used in our laboratory has one pin-hole per square inch, and must be used in lapped layers. Tests on 0.005 inch sheets have shown an ability to withstand 20 K.V. on a 2000 microsecond pulse on selected sections where there are no pin-holes. MacFayden, Edwards, and Goodwin have shown[15] that there is a systematic increase in the dielectric strength of organic liquids as the duration of the pulse voltage increases. The times involved for some liquids are of the order of a microsecond. The delay in breakdown of solids is believed to be considerably greater. Samples of 0.005 polytetrafluorethylene sheet have withstood repeated 12 microsecond pulses in excess of 40 K.V. The delay phenomenon should be of considerable help since the highest voltage is maintained for a rather short time. The experimental transformer was insulated with a double lapped layer of

0.005 inch thickness. This insulation was increased on the final transformer to three layers. Breakdown tests on the experimental transformer are described in section 3.27.

3.20. Many methods of reducing leakage inductance increase shunt capacity in the same ratio. The shunt capacity is directly proportional to the winding length while the leakage inductance is inversely proportional. Inductance is directly proportional to the thickness, while distributed capacity is inversely proportional to it. Varying either winding length or thickness will not alter the resonant frequency. In the case of the radius, while the inductance varies as $1/r^3$, the capacity varies only as the first power of r (since it depends upon the surface area of one layer). By increasing the size of the core, it should be possible to raise the resonant frequency indefinitely. We have not checked this experimentally, and are limited in our application by the choice of core sizes. When the methods at hand vary L and C so that the product is constant, a suitable design would have the secondary capacity not much less than load capacity. This gives approximately $\sqrt{2}$ x (the highest resonant frequency realizable), or, the product LC is twice the limiting value.

MEASUREMENTS ON THE EXPERIMENTAL TRANSFORMER

3.21. The resonant frequency is 24 K.C. with 140 micro-microfarads external load. Q is then about 4 (primary shorted). The resonant frequency of the winding alone is 46 K.C. with a

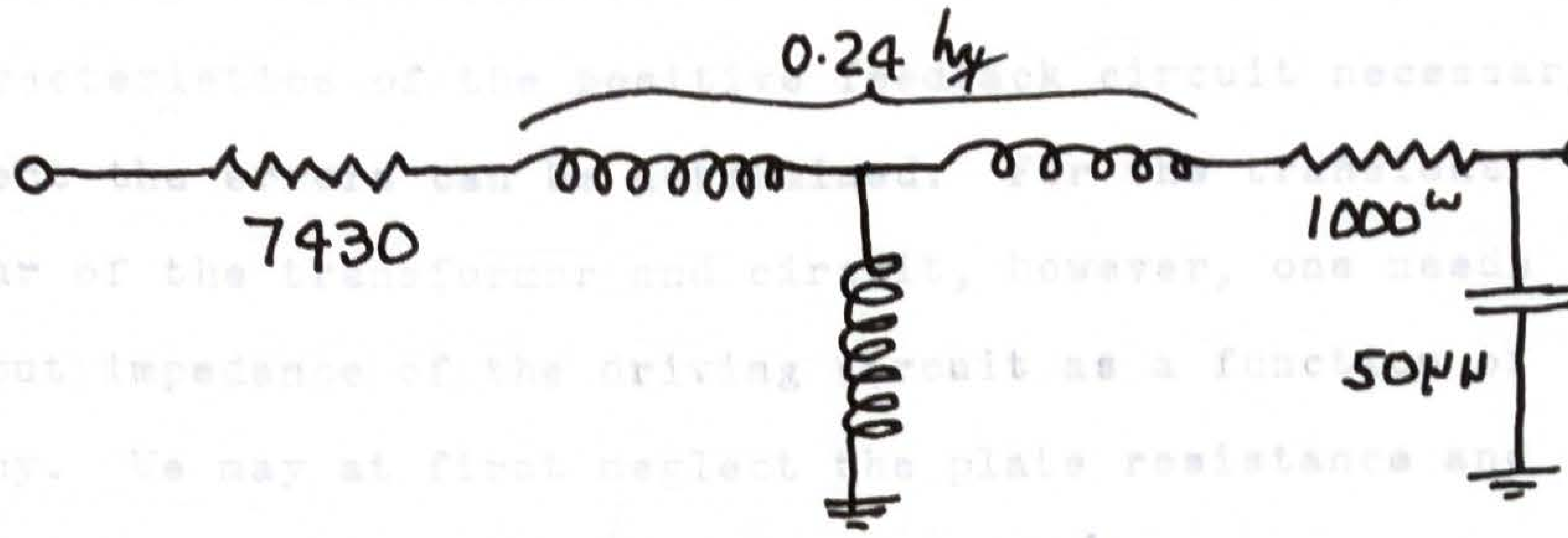


FIG 3.21.1

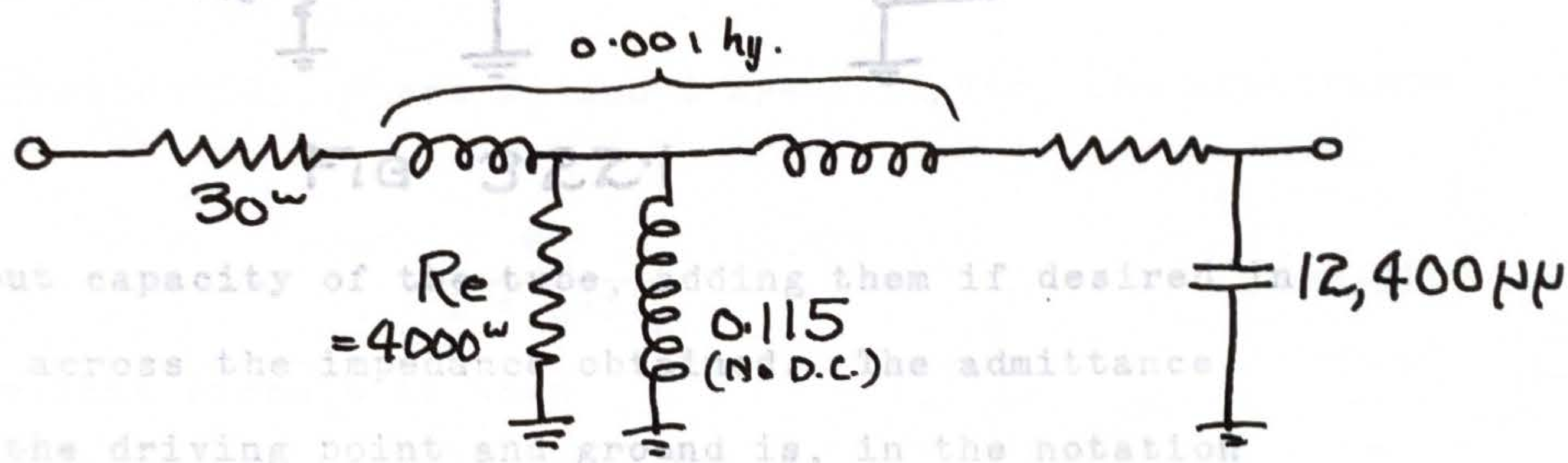


FIG 3.21.2

Q of 7.5.

IMPEDANCE OF THE DRIVING CIRCUIT

3.22. The point by point analysis of the circuit, given in sections 3.11, 3.12, 3.13, shows the sources of progressive departure of the voltage from a straight line, that is, those errors which can best be expressed as the higher order terms of a power series such as (3.12.6). Quantitative values can be

found when the characteristics of the circuit are known, and the characteristics of the positive feedback circuit necessary to correct the errors can be determined. For the transient behaviour of the transformer and circuit, however, one needs the output impedance of the driving circuit as a function of frequency. We may at first neglect the plate resistance and

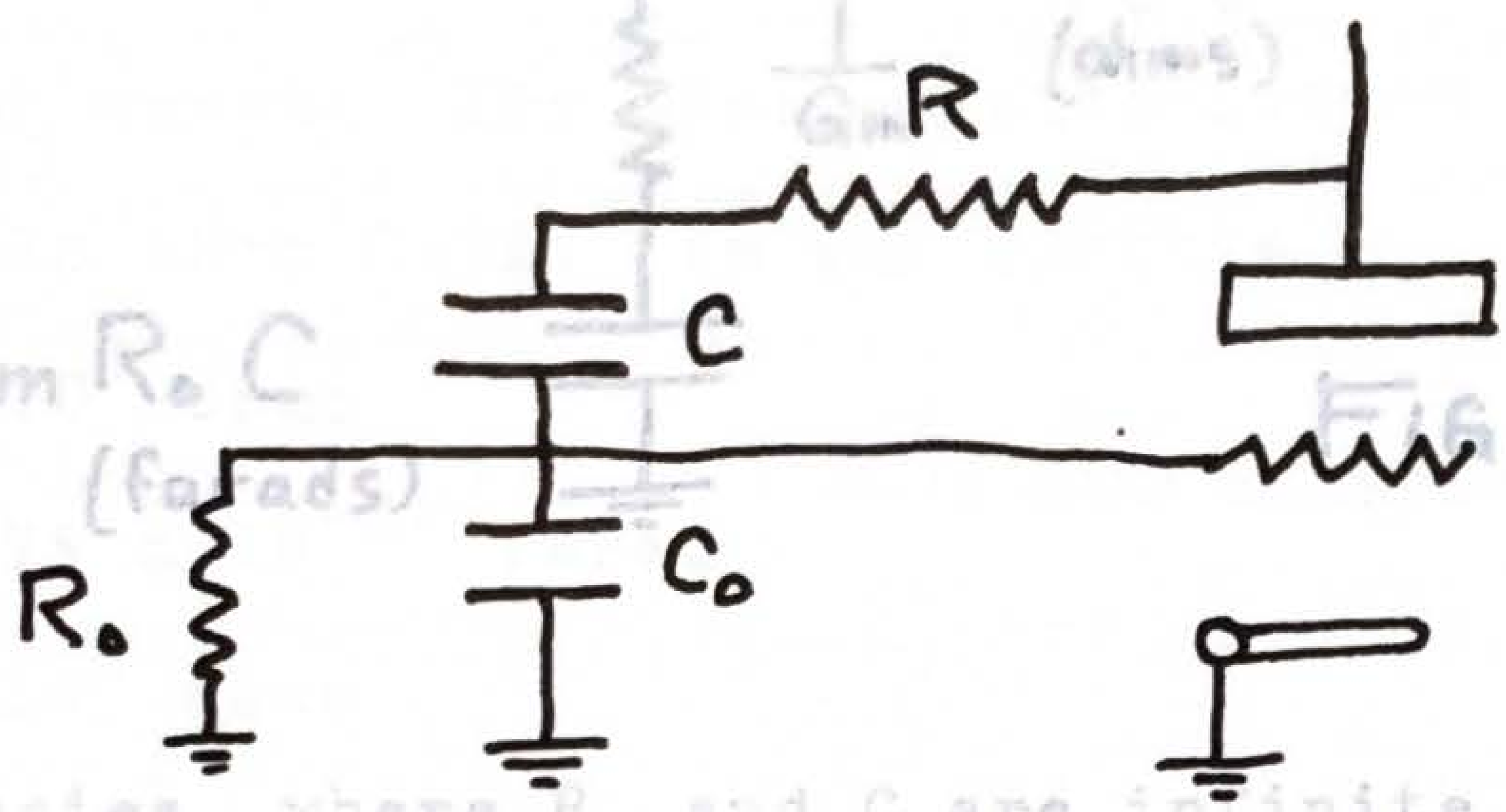


FIG 3·22·1

the output capacity of the tube, adding them if desired in parallel across the impedance obtained. The admittance between the driving point and ground is, in the notation

I_p = plate current
E_p = plate voltage
E_g = grid voltage
$I_p = G_m . E_g$

$$\frac{I_p}{E_p} = \frac{G_m E_g}{E_p}$$

$$= G_m \frac{\dfrac{R_o}{1 + j\omega R_o C_o}}{\dfrac{R_o}{1 + j\omega R_o C_o} + \dfrac{1 + j\omega RC}{j\omega C}}$$

$$= G_m \frac{j\omega C R_o}{1 - \omega^2 R_o C_o RC + j\omega(R_o C + R_o C_o + RC)} \qquad (3.22.1)$$

neglecting plate resistance and capacity. At low frequencies, where R and C_o may be considered zero, the admittance becomes

$$\frac{j\omega G_m R_o C}{1 + j\omega R_o C} . \tag{3.22.2}$$

The equivalent circuit is shown in figure 3.22.2.

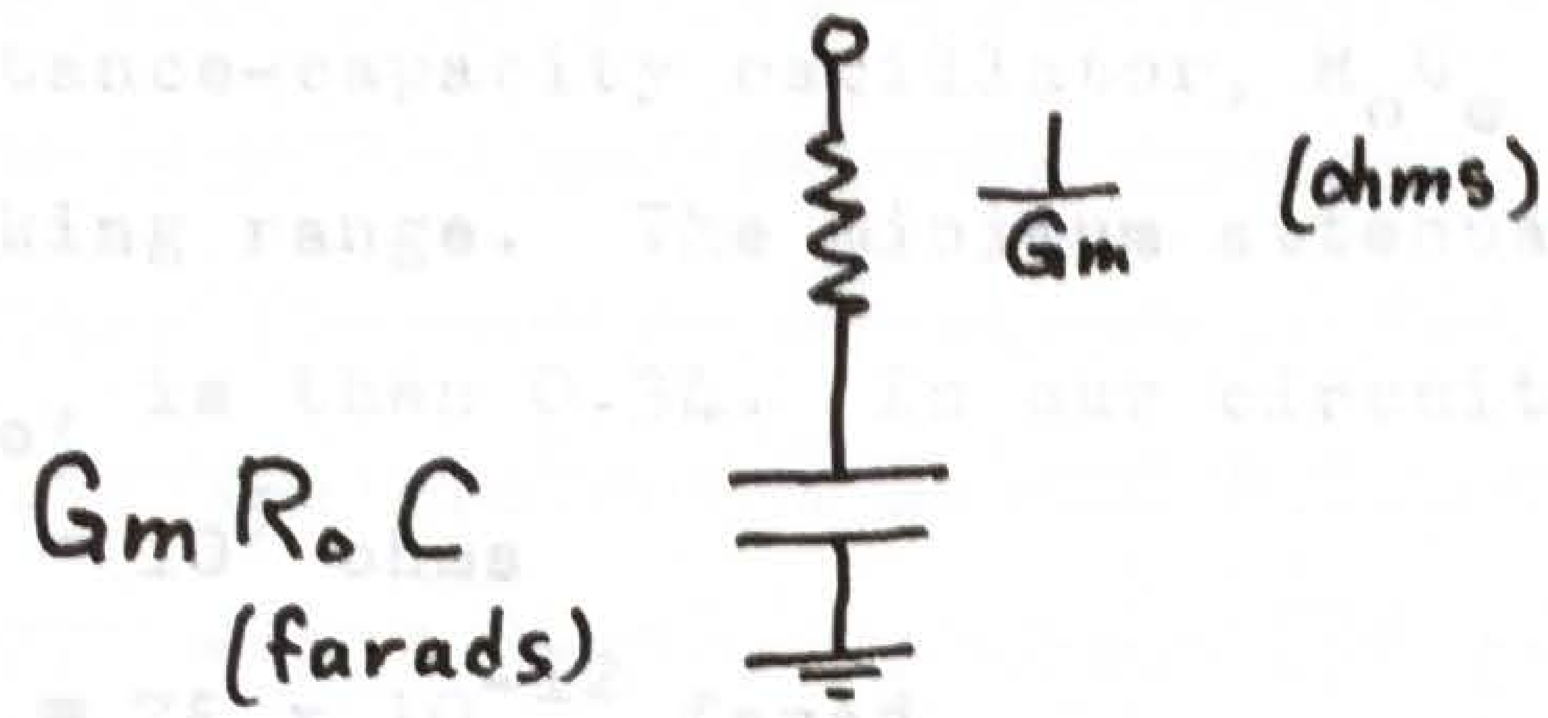


FIG. 3.22.2

At high frequencies, where R_o and C are infinite, the admittance becomes

$$\frac{G_m}{1 + jR\omega C_o} . \tag{3.22.3}$$

The equivalent circuit is then

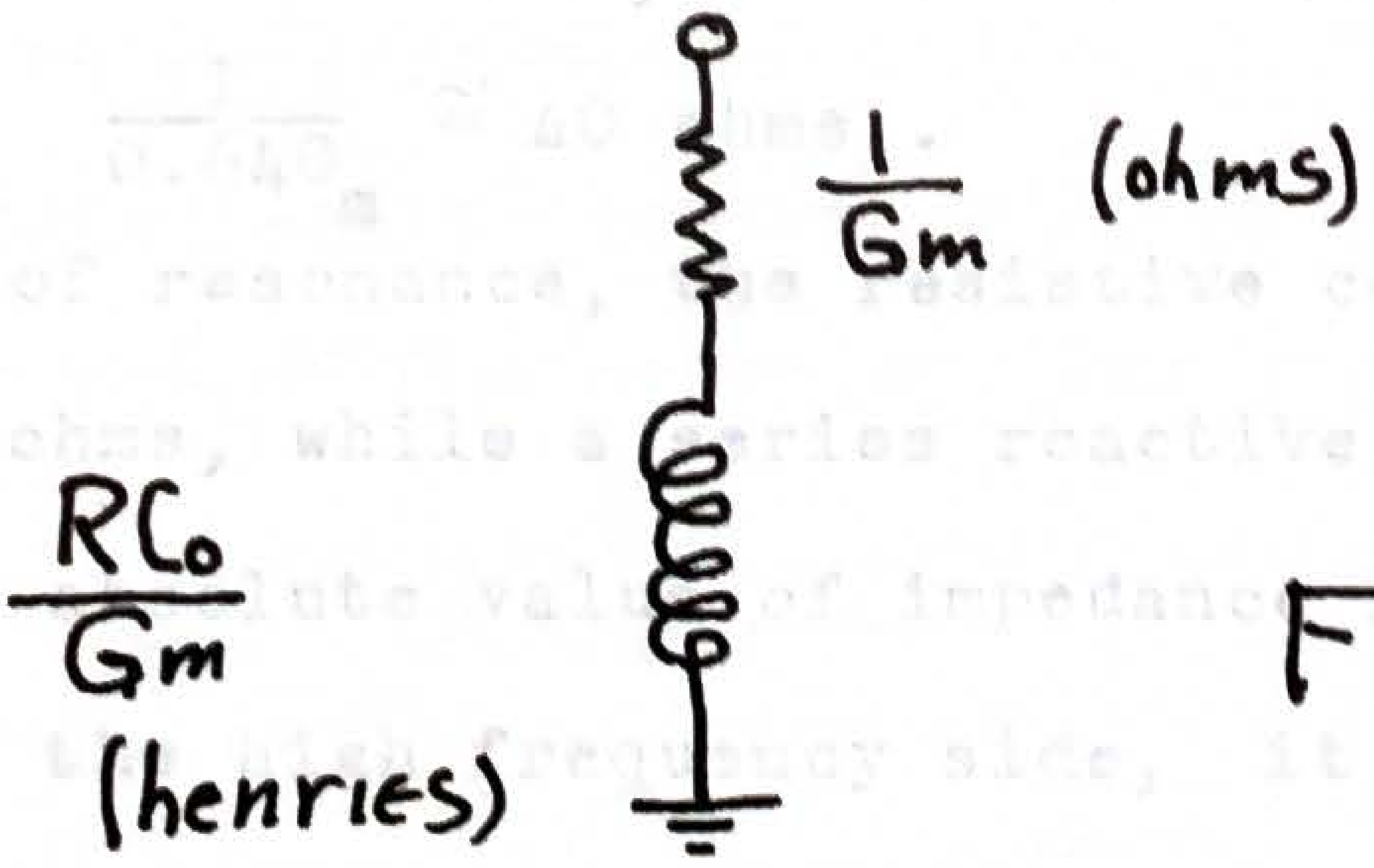


FIG. 3.22.3

Between the two extremes is a frequency where the impedance is a pure resistance. The frequency at which this occurs may be found by setting the imaginary part of the general impedance (3.22.1) equal to zero. That is,

$$\frac{(1 - \omega^2 R_o C_o RC) G_m}{(1 - \omega^2 R_o C_o RC)^2 + \omega^2 (R_o C + R_o C_o + RC)^2} = 0 \quad .$$

This yields the well known expression

$$\omega = \frac{1}{\sqrt{R_o C_o RC}} \quad . \tag{3.22.4}$$

In the resistance-capacity oscillator, $R_o C_o$ is kept equal to RC over the working range. The minimum attenuation of the combination RC, $R_o C_o$, is then 0.34. In our circuit

$$\begin{aligned} R_o &= 10^5 \text{ ohms} \\ C_o &= 75 \times 10^{-12} \text{ farad} \\ R &= 10^3 \text{ ohms} \\ C &= 10^{-9} \text{ farad} \quad . \end{aligned} \tag{3.22.5}$$

These values give a resonant frequency of 21.3 K.C. The minimum attenuation through the network is, however, 0.64. At the resonant frequency of the network, the impedance of the driving circuit is a pure resistance, and has a minimum absolute value

$$\frac{1}{0.64 G_m} \cong 40 \text{ ohms} \quad . \tag{3.22.6}$$

On either side of resonance, the resistive component decreases toward $\frac{1}{G_m} = 25$ ohms, while a series reactive element appears which makes the absolute value of impedance increase in both directions. On the high frequency side, it is inductive, on the low frequency side, capacitive (figures 3.22.2 and 3.22.3). In considering the transient response of the circuit, the resonant value of impedance may be used, since the resonance is quite broad, although the loaded transformer resonance does not coincide exactly with that of the circuit.

ADDING THE RISING VOLTAGE TO A 500 K.V. SOURCE

3.23. The design of the coupling circuit between the transformer and the apparatus used to supply a 500 K.V. source depends critically upon the minimum value of the sparkover protection resistor. High voltage acceleration column workers have found by experience that high voltage condensers discharged by an accidental arc can do considerable damage. A sizeable hole is sometimes melted through an electrode. The energy in a typical condenser, 0.0033 microfarads at 500 K.V., is

$$\tfrac{1}{2}CV^2 = 417 \text{ joules or about } 100 \text{ calories .}$$

Taking the average specific heat of iron as 0.16, one gram of iron requires about 240 calories of heat to raise it to the melting point. Considering the amount of energy available, the transfer between the unfortunate electrode and the condenser must be quite efficient. The function of the spark protection resistor is to cause the energy to be developed in the resistor instead of in the apparatus.

3.24. The protective resistors which have been in use in this laboratory are of the order of one megohm. It is, of course,

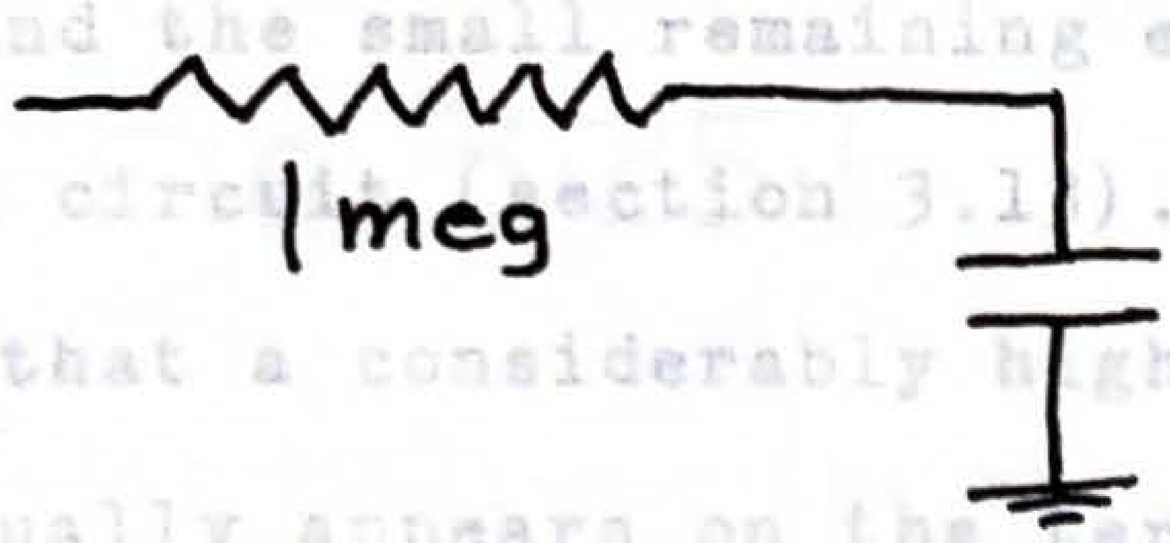


FIG 3.24.1

impossible to apply a rising voltage of 90 M.V.·sec^{-1} through a resistor of 1 megohm, since the steady drop is 12.6 K.V. and the time constant 140 microseconds (section 3.17). A possible

solution was to use a condenser in a parallel circuit (figure 3.24.2), the condenser being so small that the energy that could be delivered during a spark would not be objectionable. In the absence of any quantitative data, one could only make the energy delivered during a spark not more than 3/2 or 2 times the value it would have without the coupling circuit. The energy in the terminal capacity to ground (140 micromikes) is about 1/20 of the energy in the high voltage condenser. The feed condenser could then be 70 to 140 micromikes. The resistor R

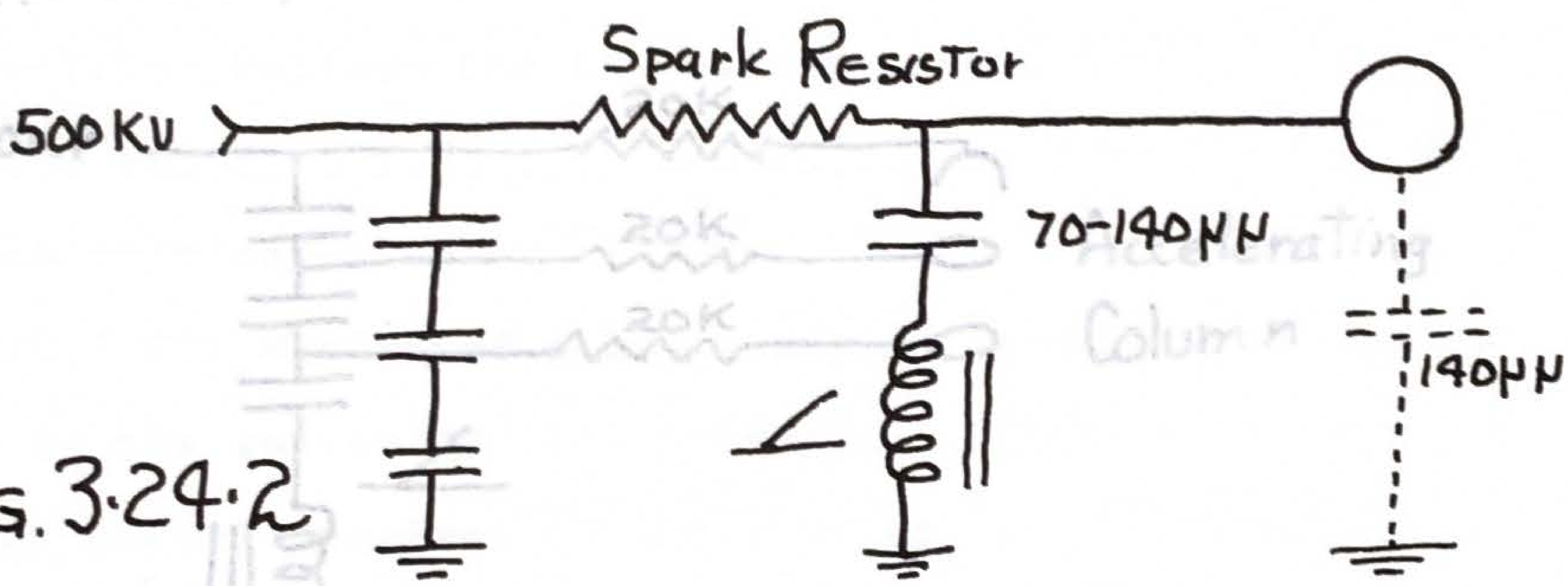


FIG. 3·24·2

now causes a type of distortion opposite to that in the circuit of figure 3.24.1. The value of R may be increased until the error is small, and the small remaining error corrected in the positive feedback circuit (section 3.13). The circuit still has the disadvantage that a considerably higher voltage must be supplied than actually appears on the terminals. For a 70 micromike feed condenser, the transformer voltage must be three times the useful value. There are the further disadvantages that the magnitude of the rise depends upon the terminal capacity, and that special arrangements must be made to vary the focusing voltages during the rise. For these reasons, this method was discarded in favour of the following one.

3.25. By lowering the spark resistor to 20,000 ohms, (in each section) the main time constant (figure 3.24.1) is reduced to 2.8 microseconds, and the voltage can now be applied to the bottom of the condenser stack (figure 3.25.1). No adjustments will now be required to the focusing electrodes since the whole difference of potential is applied to the beam after acceleration to 2/3 of its final voltage. Peak currents during an arc cannot be greater than 10 to 30 amps, a fairly innocuous value. It seems very likely that the arc resistance could never approach 20,000 ohms.

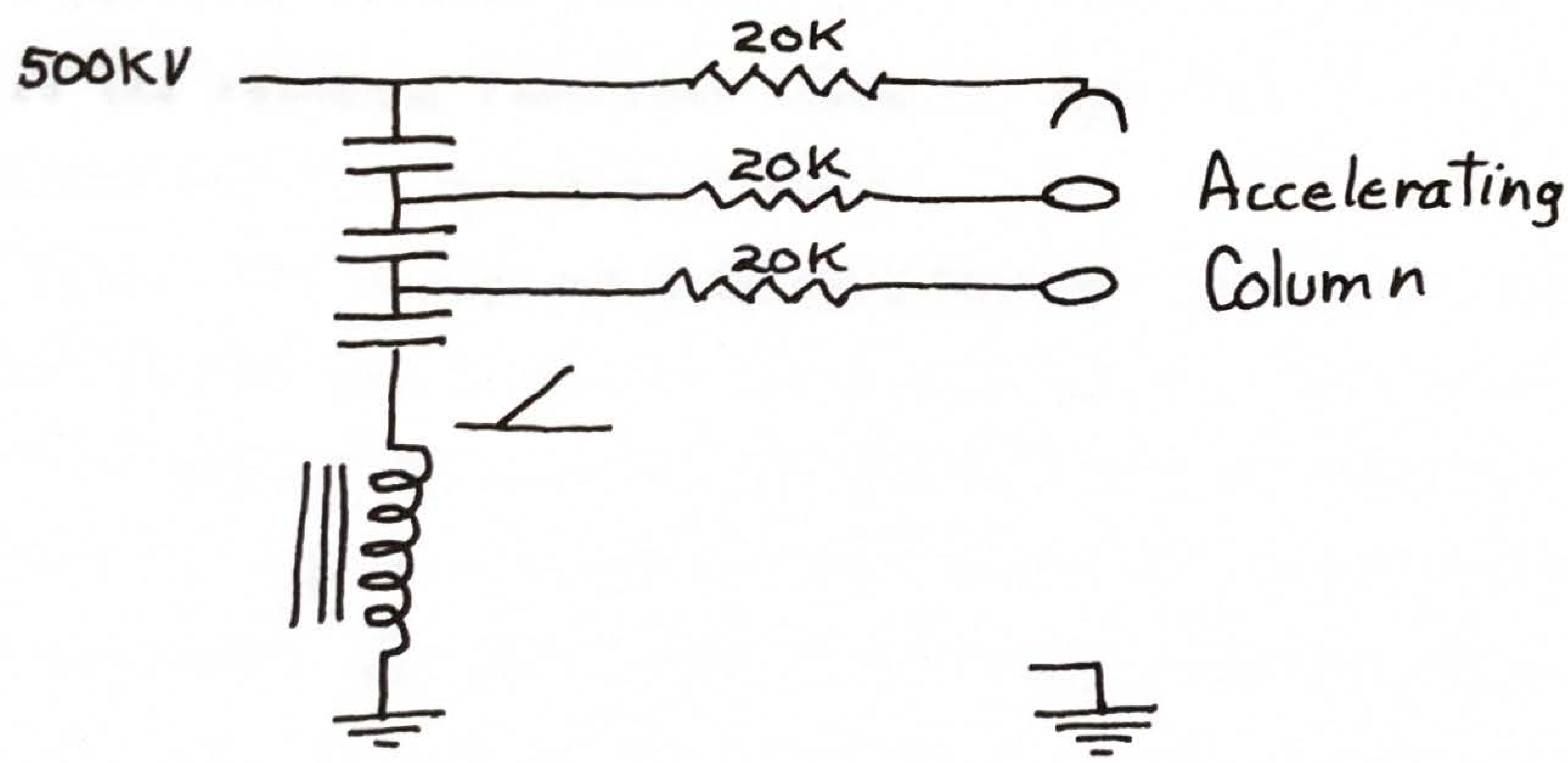


FIG. 3·25·1

The equivalent circuit involving the redesigned transformer is shown in figure 3.25.2. If the spark resistor is omitted, the critical resistance in the circuit is

$$r = 2\sqrt{\frac{L}{C}} = 40,000 \text{ ohms}, \qquad (3.25.1)$$

and the resonant frequency is 33 kilocycles. Inserting the 20 K

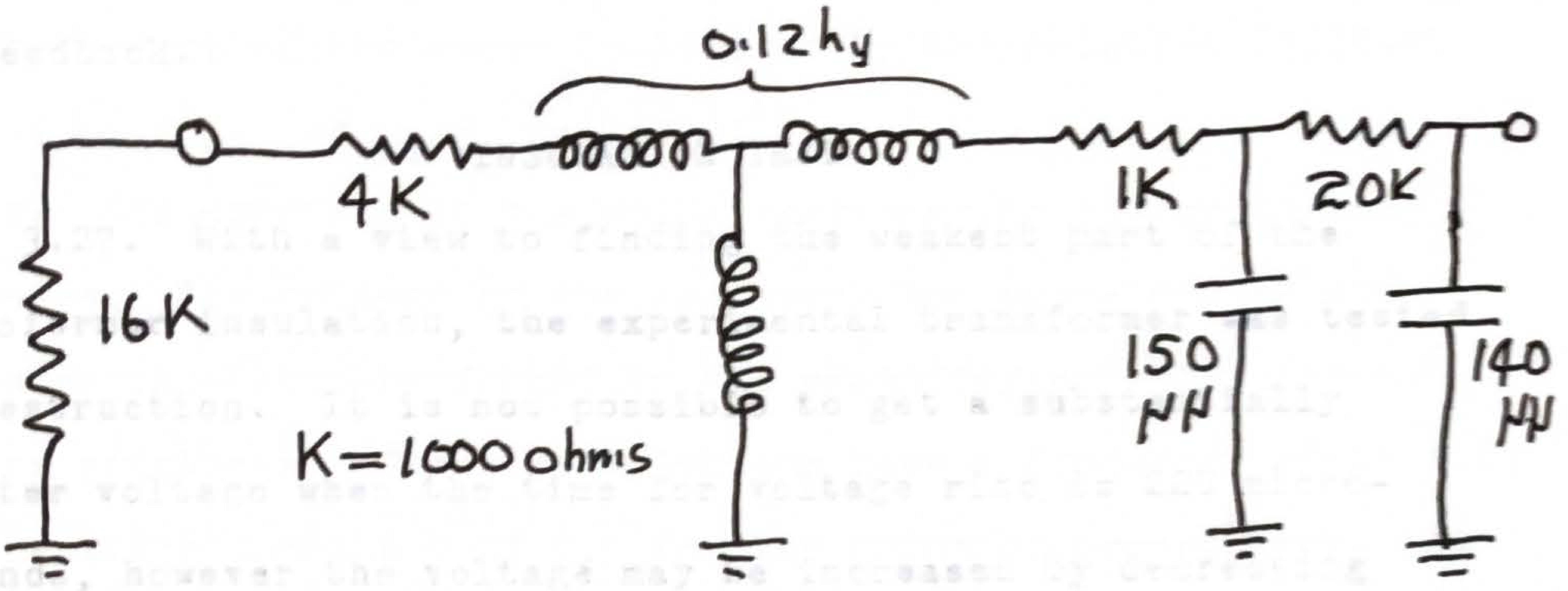


FIG. 3.25.2

spark resistor between the two halves of the total capacity, raises the resonant frequency somewhat, and does not quite establish critical damping.

3.26. The design of a transformer has been discussed with respect to the choice of core material, maximum flux density, amount of copper required in the winding, and disposal of windings for the most suitable combination of leakage reactance and shunt capacity. The effect of leakage reactance and shunt capacity on the output wave form has been shown and the form of coupling circuit which gives the closest approach to the desired wave form has been found. It has been shown that the transformer and output circuit can be designed to give an output voltage which approaches quickly a linear rise when the primary is properly driven, and that the most convenient driving circuit is a single stage Miller circuit. When the flux density in the core is not too high, the circuit errors have been shown to consist chiefly of two higher order terms with coefficients of the order of a few per cent. These errors can be corrected

by feedback.

INSULATION TEST

3.27. With a view to finding the weakest part of the transformer insulation, the experimental transformer was tested to destruction. It is not possible to get a substantially greater voltage when the time for voltage rise is 220 micro-seconds, however the voltage may be increased by decreasing the length of the induced pulse. If a charged condenser is suddenly connected to the primary, the primary wave form for low amplitudes is shown in figure 3.27.1. The frequency is

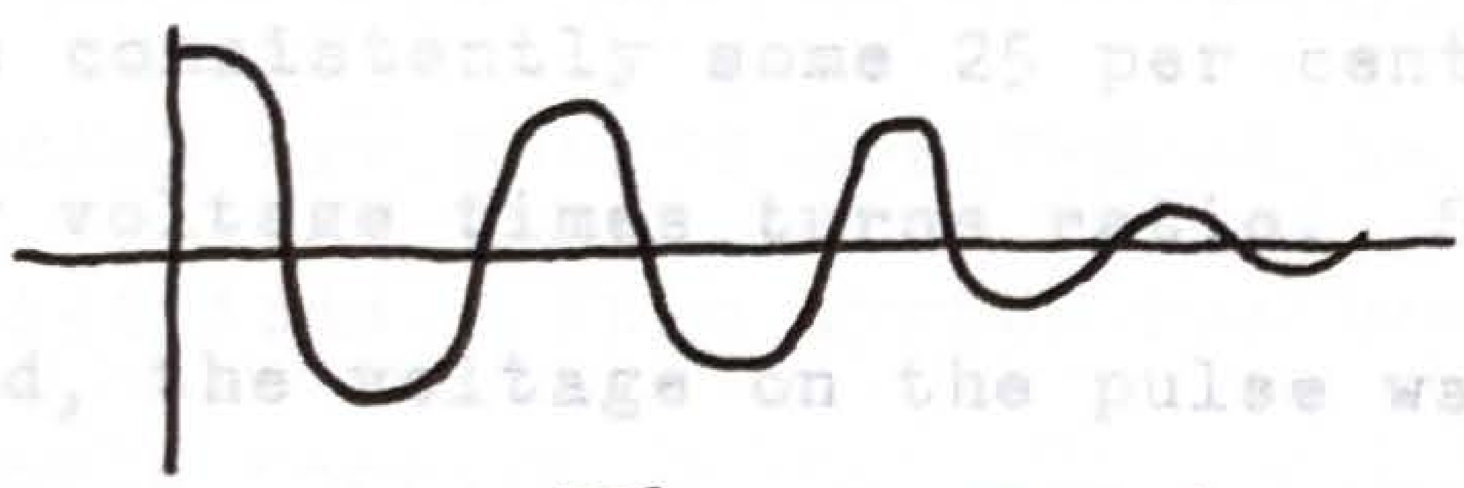


FIG. 3.27.1

660 cycles per second. For large amplitudes, the area under

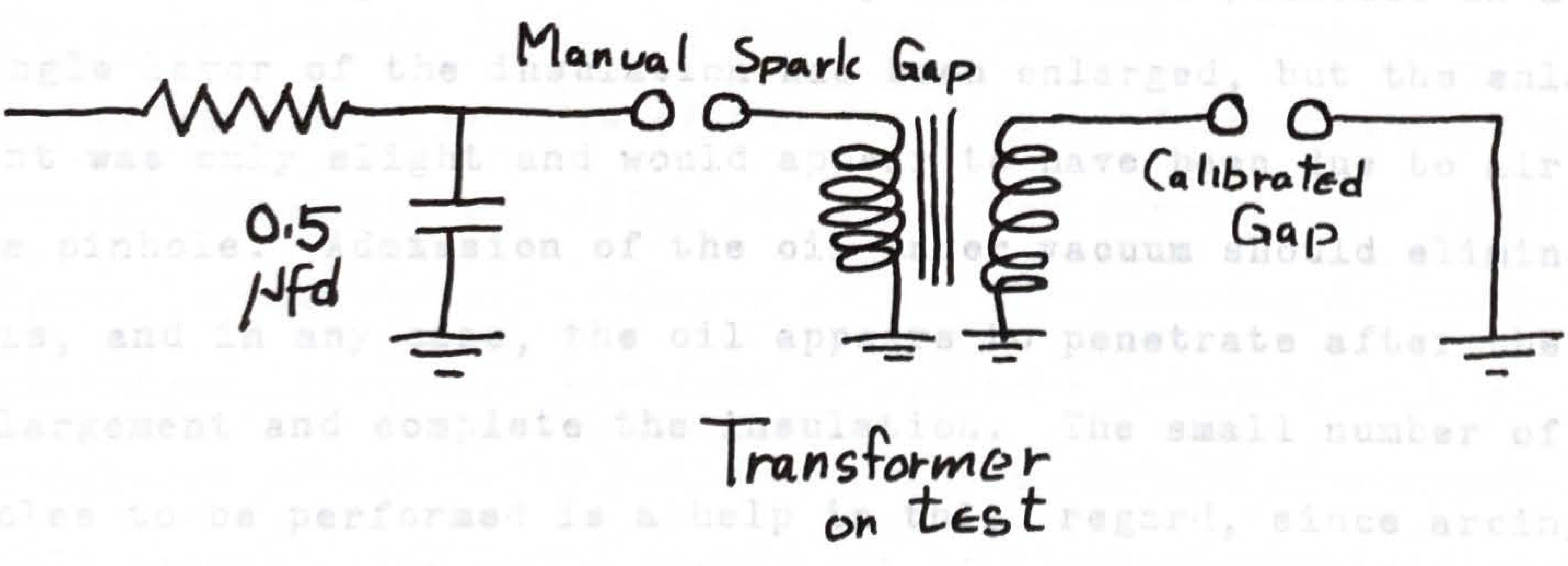


FIG 3.27.2

the first part of the curve is limited by the relation (3.10.2)

$$\phi_{max} = \frac{10^8}{N}\left[\int E \, dt\right]_{max}.$$

That is, the duration of the pulse decreases as the amplitude increases. The voltage on the secondary has superimposed upon it the oscillations performed by the secondary capacity and the leakage reactance. These oscillations have a frequency of 46 K.C; and the circuit has a Q of 7.5 for the unloaded transformer. At 100 K.V. the duration of the pulse is about 70 microseconds, that is, time for about three oscillations. These oscillations increase the effective secondary voltage to a value considerably above that indicated by the turns ratio. The voltage indicated by the gap was consistently some 25 per cent higher than the product of primary voltage times turns ratio. Since the calibration for DC was used, the voltage on the pulse was certainly higher than that indicated by the gap. Failure occurred at 90 K.V. on the gap, and the transformer was unwound and examined layer by layer. At least one layer was intact everywhere. Some pinholes in a single layer of the insulation had been enlarged, but the enlargement was only slight and would appear to have been due to air in the pinhole. Admission of the oil under vacuum should eliminate this, and in any case, the oil appears to penetrate after the enlargement and complete the insulation. The small number of cycles to be performed is a help in this regard, since arcing in an air enclosure is followed by a period of time great enough to allow oil to reach the insulation. The failure was found to be in an experimental feedback winding wound over the primary

with no insulation and displaced axially. Since this winding is not part of the final design, the test does not show where the weakest point in the insulation is. However, since failure occurred at over three times the working voltage, it does show that the insulation is adequate for the design voltage.

STABILIZING A 500 K.V. SOURCE

3.28. To guarantee that the relation (3.5.1) is maintained sufficiently closely to keep the amplitude of the betatron oscillations at injection down to a few millimeters, it is necessary to stabilize a 500 K.V. source to 1/10 per cent. The failings of the methods which had been tried will be indicated, before discussing the method arrived at as a result of the writer's investigations. This system has been in successful operation for over a year.

3.29. The difficulties of stabilizing voltages of 500 K.V. to 1/10 per cent are chiefly associated with the magnitude of the electrostatic fields encountered. Because of the space limitations, and because of the difficulty of keeping surfaces smooth, gradients exceeding the value for corona occur at many places in the system. Because of the irregularity of the corona discharges, insulated sections or sections isolated by high impedances occasionally build up charges and are discharged by small sparks. The small sparks set up electromagnetic disturbances as well as electrostatic disturbances which are very troublesome. Because of space limitations again, complete electrostatic screening of parts is seldom possible. Measuring

resistors have often very small irregular discharges from them and also pick up disturbances occurring elsewhere. The result of all these effects is that whenever an amplifier is included in the high voltage system, or when the impedance is raised by the insertion of controlling elements, extra disturbances appear. The fact that condenser-rectifier sets can have fairly low output impedances is one reason why more trouble-free operation is obtained from them than from high impedance generators such as the Van de Graaff. The presence of high voltages of mains frequency also causes trouble. Where a many-section filter is used, the electrostatic induction on the final terminal may exceed the transmitted ripple many times. The reason for this is easily seen from the usual layout in which one condenser stack carries an alternating voltage of from 30 to 100 K.V. An inter-capacity between the large output terminal and the A.C. stack of only a few micromicrofarads then causes a voltage of from 60 to 200 volts in the output, when the output condenser is 0.001 microfarads. Electrostatic induction between transformer primary and secondary also causes trouble when it is desired to add a subsidiary voltage to the system for regulation. All these effects are due primarily to the fact that the size of the components is not scaled up according to the voltage, a fact which may be appreciated by comparing the volume occupied by a 500 K.V. set with that allotted to a 1000 volt power supply.

METHODS CURRENTLY USED FOR H.T. STABILIZATION

3.30. The most frequently tried method of stabilizing rectifying sets is the insertion of a variable power supply

between the bottom of the rectifier set and ground. The first difficulty encountered is the large A.C. capacity current to ground. This constitutes the chief load in most rectifier sets, and is usually at least ten times the useful current generated by the set. It can be eliminated by shielding the transformer. This method has been developed by Preston at Oxford. Preston's stabilizer contains, in essence, a 3-valve amplifier A.C. coupled to the output of the set. The amplifier feeds a variable voltage of 5 - 10 K.V. into the output through the normally grounded terminal. When oscillations had been suppressed, it was found that the expected decrease of about 50 times in A.C. ripple had been achieved, but a new irregular disturbance was being introduced which reduced the net improvement to a factor of 3 or 4. It seems very likely that this was due to the combined effects of pickup in the amplifier and corona current drop across the amplifier impedance. Experience with this circuit at the Clarendon laboratory has led to its abandonment. An extra filter section is used instead. D.C. stabilization is achieved in effect by using a galvanometer and photocell to turn on the counting circuits when the voltage reaches a predetermined value. In the Cambridge high voltage sets the need for stabilizing the voltage is circumvented by the use of an analyzer. The arc source provides a wide spread of particle energies (centred on 10 K.V.) and the unfiltered multiplier set has 30 K.V. ripple. The rectifier set in the physics laboratory of the National Research Council at Ottawa uses a motor generator set with a synchronous motor to provide a steady A.C.

supply. Mains frequency variations are not more than 1/3 per cent where this set is installed. The mains frequency in Birmingham is not sufficiently steady to make this method useful. While many other schemes, such as reverse current tubes, have been suggested and tried, we do not know of any which are actually in use at present.

METHODS WHICH CAN BE APPLIED TO THE PRESENT PROBLEM

3.31. None of the stabilizing methods described above can be applied to the present problem. A number of possible methods will now be described briefly. An auxiliary rectifier could be placed at the high voltage end of the H.T. set and controlled from the difference between the overall voltage and some standard of reference. Placing the rectifier at the high end of the H.T. supply eliminates the trouble due to capacity currents and the use of R.F. makes possible a convenient connection between the low end and the high end. Centimeter wave and optical links between a control tube at the top and an auxiliary circuit at ground are possible. A circuit containing a two tube amplifier at the high end has many attractive features and has been considered in detail. It is based on the use of two tubes in series, an arrangement which was applied to electron microscope stabilizers by Philips.

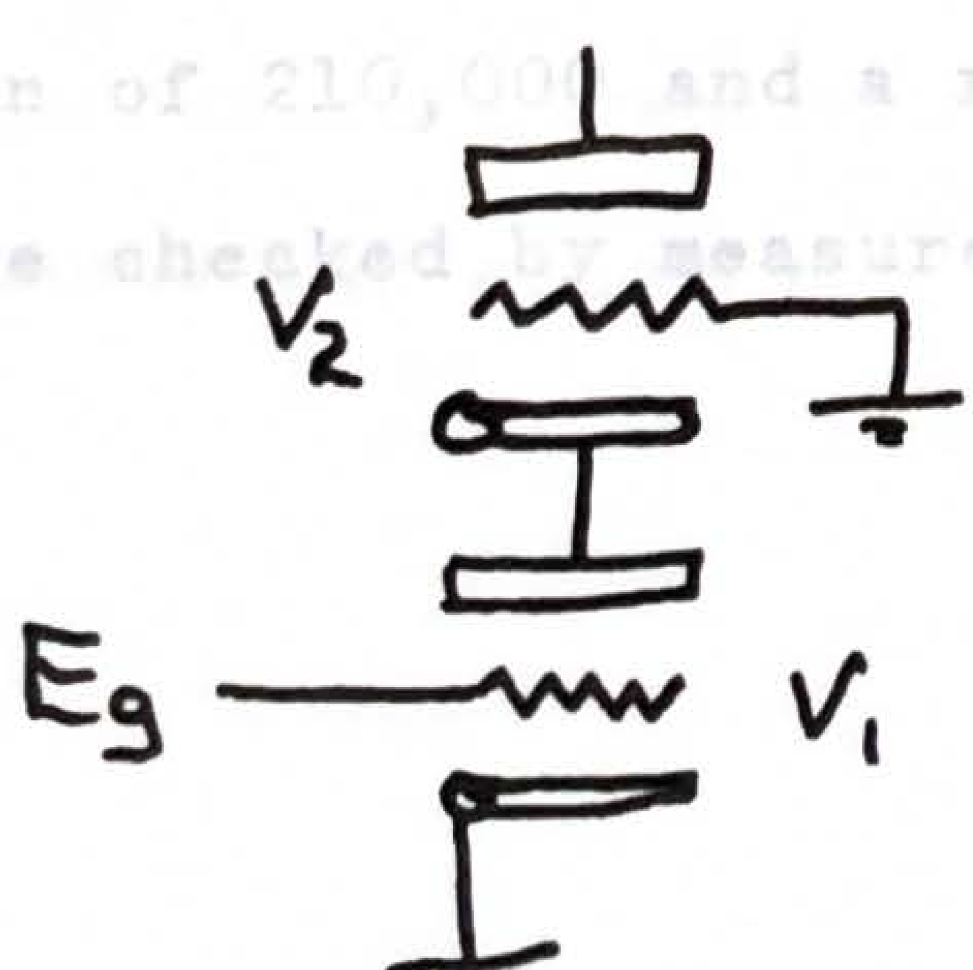


FIG. 3.31.1

The circuit can be reduced to

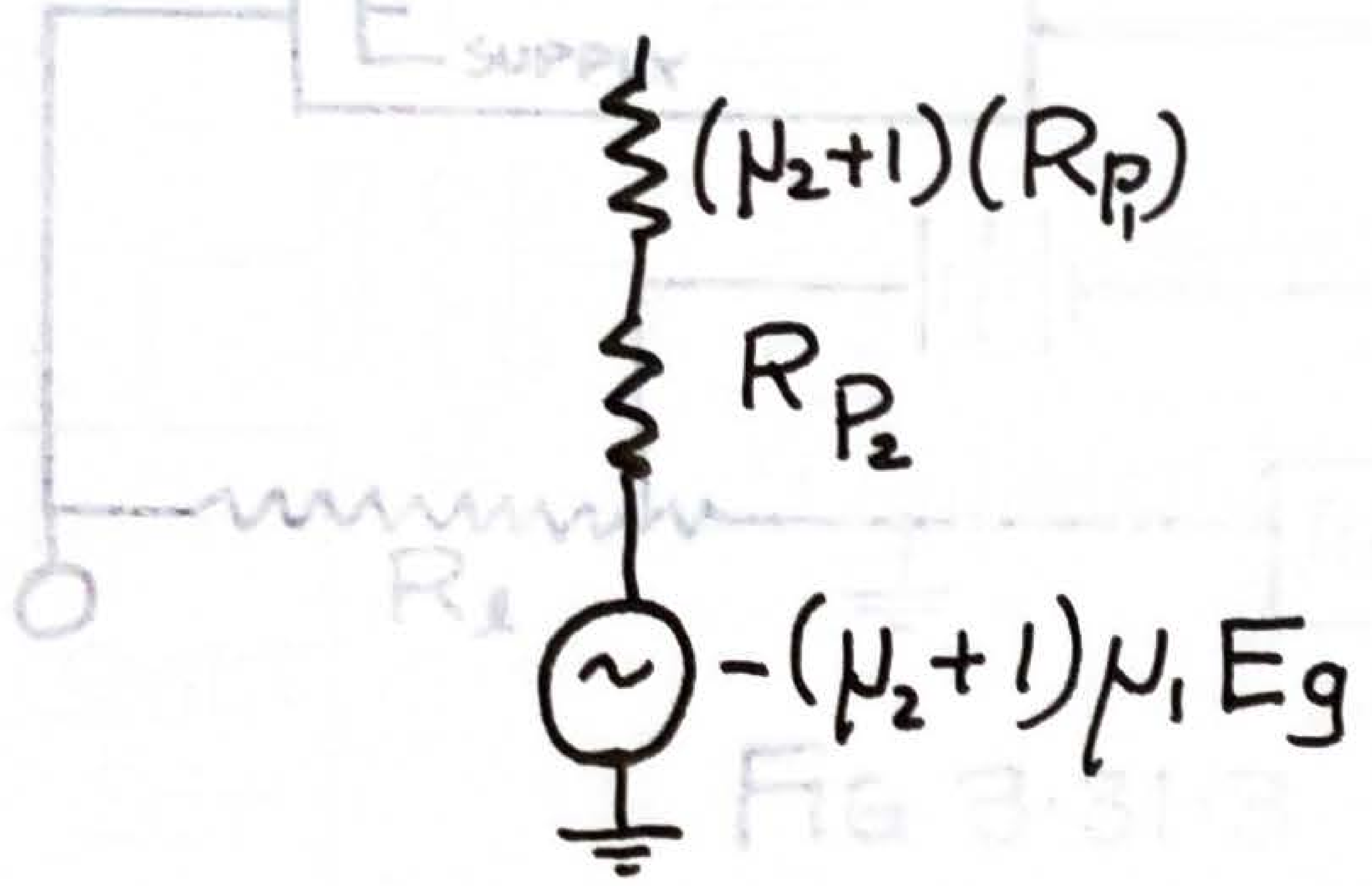


FIG 3.31.2

That is, the whole amplifier has a gain of

$$G = \mu_1(\mu_2 + 1) \tag{3.31.1}$$

when operated with a constant current source, and an impedance of

$$R = R_{p_2} + (\mu_2 + 1)R_{p_1} \quad . \tag{3.31.2}$$

When currents of about 1 milliampere are required, V_1 may be a small high-gain pentode, and V_2 a power triode with a high voltage rating and as high a μ as possible. The plate dissipation required at 20 K.V. is only 20 watts, therefore a small tube such as one of the Eimac series will do. Eimacs have excellent characteristics for high voltage work, since the internal structure is simple, the spacings are large, and the tantalum elements make it easy to maintain a high vacuum. The measured constants of an Eimac 250TH, and an EF91 pentode, gave a gain of 210,000 and a resistance of 216 megohms. These values were checked by measurement on an amplifier. When the

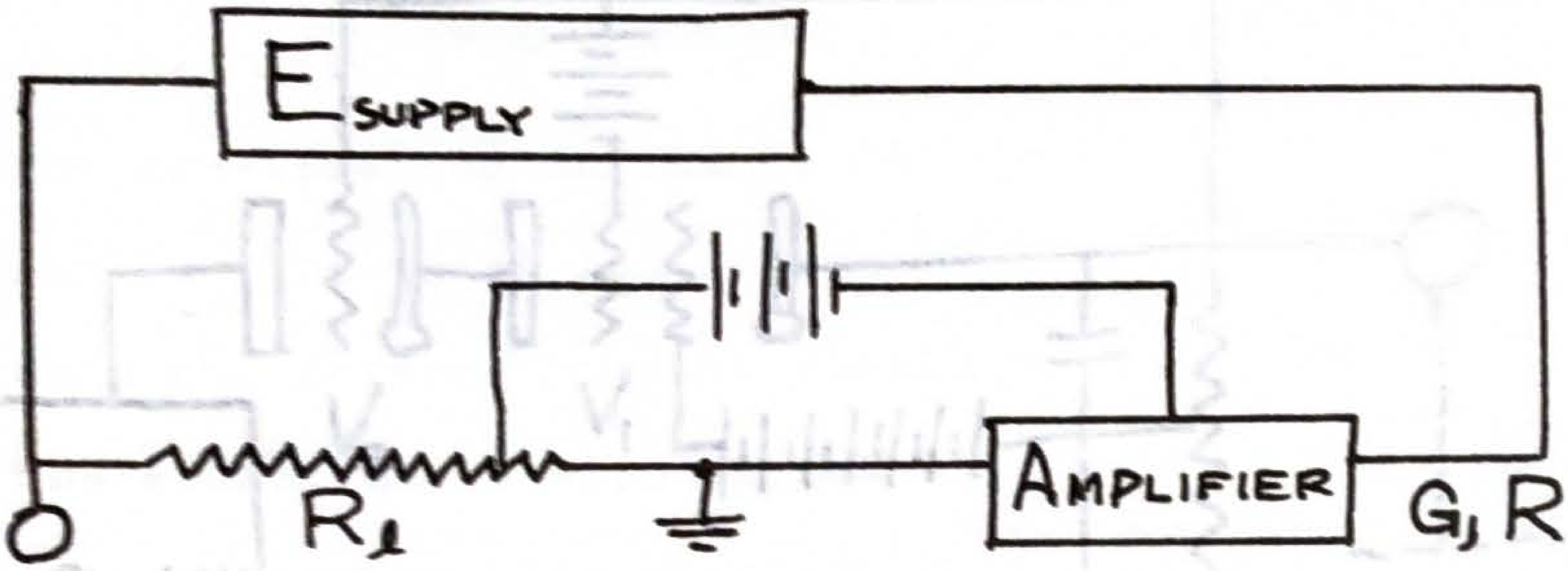


FIG 3.31.3

p = divider ratio

R_ℓ = divider resistance

G = voltage gain of the amplifier

R = internal resistance of the amplifier

amplifier is used in a stabilizer, the stabilization ratio is

$$\frac{\partial E_o}{\partial E_s} = 1 + pG + \frac{R}{R_\ell} \quad . \tag{3.31.3}$$

(The term R/R_ℓ arises from the fact that the ratio was defined as a comparison of absolute magnitudes.) The output impedance is

$$\frac{RR_\ell}{R_\ell(1 + pG) + R} \quad . \tag{3.31.4}$$

That is, the amplifier resistance is reduced in approximately the ratio of the feedback factor. For the 500 K.V. stabilizer, if the reference voltage were 500 volts, p = 1000, hence the stabilization ratio is 210 and internal resistance is approximately 1 megohm. In view of the general considerations in regard to pickup and impedance given in section 3.29, and because of the extra components and extra power required at the H.T. end,

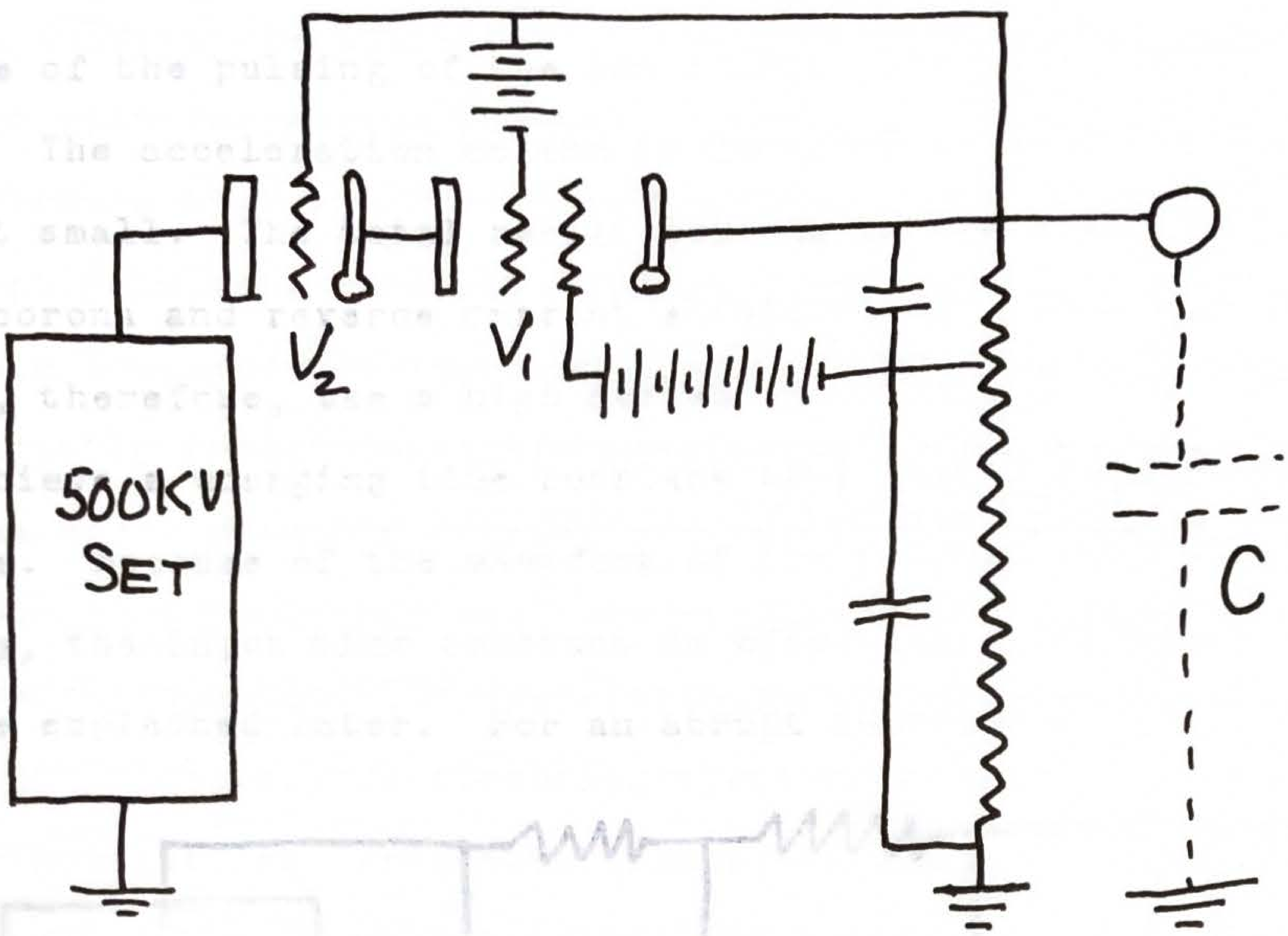


FIG. 3·31·4

this circuit is considered less useful than the simpler one to follow, although it provides the highest combination of stabilization factor and speed that can be obtained simply.

STABILIZATION WITH A SATURABLE REACTOR

3.32. In the method to be considered finally, a saturable reactor is placed in the primary of the transformer of the rectifier set and controlled through a feedback loop by the output voltage measured by suitable means. This method has the advantage of retaining the simplicity and good output impedance of the high voltage circuit of the condenser-rectifier set, but is limited in speed. The importance of this limitation depends upon the magnitude of the load current. In our case,

because of the pulsing of the ion source, the load current is small. The acceleration column is designed to make the electron current small. The total resistance due to the R.F. transmission line, corona and reverse current should be more than 1000 megohms. We may, therefore, use a high series resistance in the filter and achieve a charging time constant of 5 tenths of a second or more. Because of the wave form of the voltage applied to the primary, the input time constant is effectively increased. This will be explained later. For an abrupt increase of input of

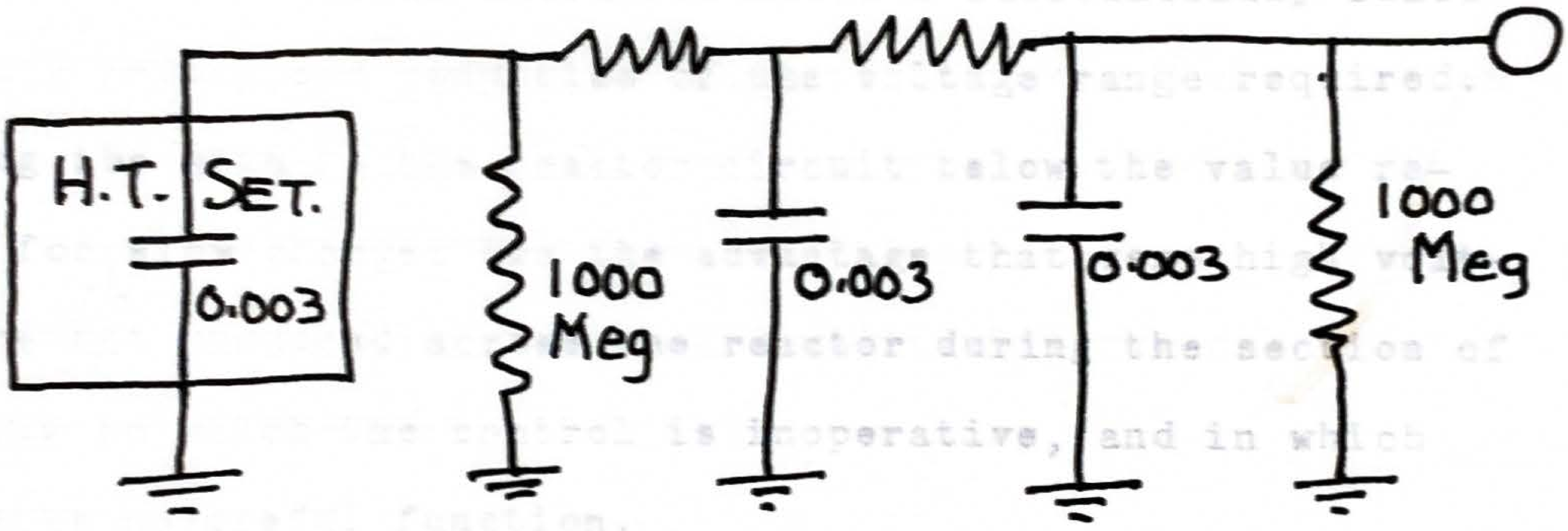


FIG. 3.32.1

10 per cent, the change in output exceeds the permissible amount by a factor of 2 or 3. Such a sudden change on a large mains circuit could be caused only by an abrupt disconnection of a large fraction of the load, and we can regard this as improbable. No abrupt changes of any magnitude have yet been observed. The discharge time constant is about 4 seconds.

3.33. To deal with 10 per cent changes of supply, the gain around the feedback loop needs to be at least 100. Since the speed of action obtained makes this gain useful for slow

changes only, it is advisable to have the gain around the loop increase with decreasing frequency. By separating the control action into a high speed part with low gain and a low speed part with high gain, the feedback problems are greatly reduced. The high gain low speed section may then consist of a motor operated variable transformer which may be made to cover as wide a range as could possibly be required to correct line voltage changes. This range may be increased to include manual output adjustments as well. The advantages to the reactor circuit of such a division include operation about a substantially fixed operating point, and reduction of the voltage range required. Reducing the gain in the reactor circuit below the value required for slow changes has the advantage that very high voltages are not produced across the reactor during the section of the cycle in which the control is inoperative, and in which they serve no useful function.

THE MEASURING ELEMENT

3.34. Tubes of low conductivity liquids used as potential dividers are subject to changes due to small amounts of impurities. Home-made resistor stacks usually break down in sections due to sharp points or unreliable dielectric supports. Pressurized Van de Graaff generators, because of the limited current supply and limited space, always use electrostatic generators to measure the voltage. These require careful protection from corona disturbances and careful shielding, and are not as reliable as they would be at lower voltages. The Philips company supplies a high precision resistor consisting of a coil of

resistance wire with a low temperature coefficient, helically wound on a cotton core. This is in turn wound on a glass cylinder, 12 cms. in diameter, which is mounted inside a vertical column. These resistors are apt to show irregular deviations in the resistance due to the opening of the 15 micron thick wire. Philips have advised the use of their "metallized" precision resistor. The resistor has a poor temperature coefficient, but is mounted in a column with forced oil circulation which is closely thermostatted. The reference voltage is obtained from voltage regulator tubes 85A1. These are designed for precision work and have unusually stable characteristics. The variation of voltage (at constant current of the rated value) is 0.2 per cent for the first 300 hours and 0.1 per cent thereafter. The temperature coefficient is -1.8 millivolts per degree Fahrenheit, and thermostatting is unnecessary.

THE AMPLIFIER

3.35. The amplifier which supplies the D.C. winding on the saturable reactor should be linear over as wide a range as possible so that the current through the reactor winding will be proportional to the average value of the input. The output impedance should be high so that the line constant of the circuit including the D.C. reactor winding will be small. Inverse feedback has been used in the preamplifier and in the final stage to accomplish these ends.

SATURABLE REACTOR

3.36. A saturable reactor is an iron cored inductor in which an auxiliary D.C. winding is used to change the permeability of the core and the inductance of the inductor. The

initial experiments on controlling voltage with such a device were done using a resistance load instead of the rectifier set. When operations were transferred to the rectifier set, some special effects were found. The normal direction of control was reversed, that is, the rectifier output increased when the

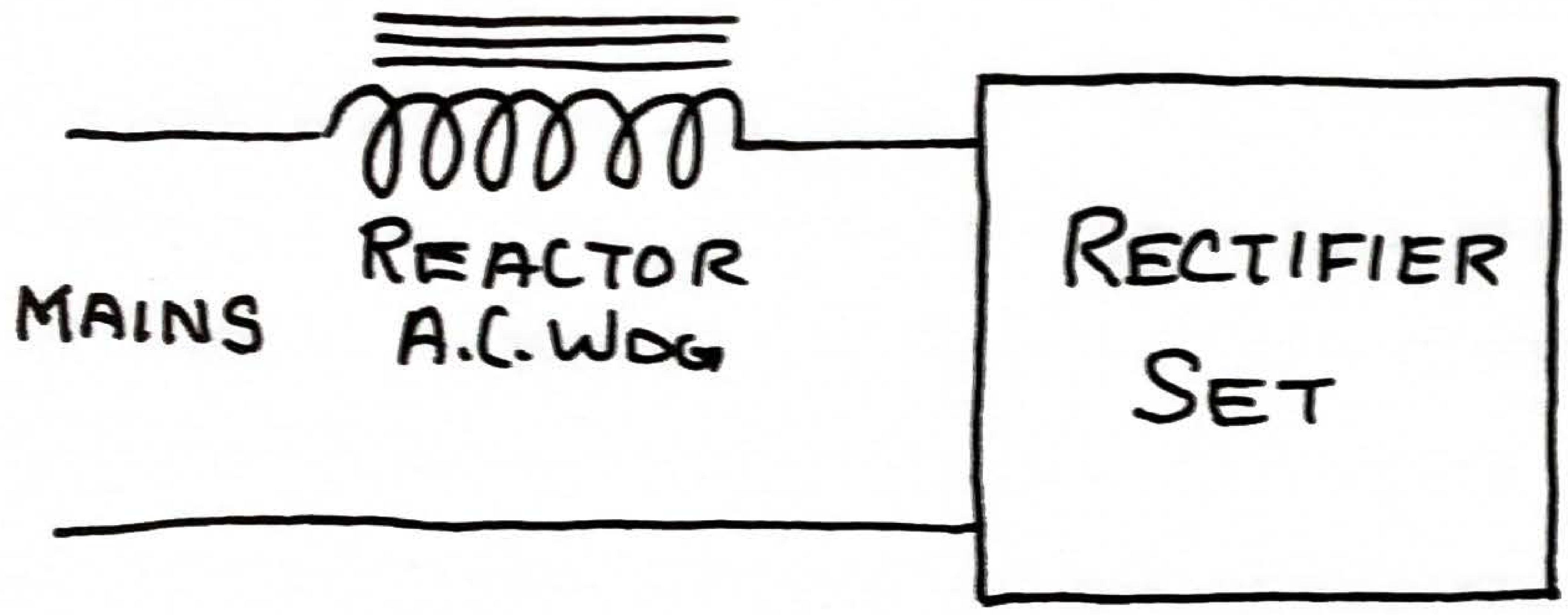


FIG 3·36

impedance of the series reactor was increased. This was because the condition of the core which produced a high impedance in the winding led to a sharply peaked wave form. The normal direction of control could be re-established by filtering the output to remove the third harmonic. A further effect was a hunting which took place at certain core biases. This was due to a dependence of extent of peaking on the A.C. current through the core. The final value of rectifier set output obtained during the charging period was higher than could be maintained after the output condensers had been charged with the corresponding reduction in primary current. The opposite effect took place when the condensers discharged.

PEAKED WAVE FORM

3.37. Another phenomenon which had not been foreseen was the effect of the peaked wave form, produced by the reactor under certain conditions, upon the transformer rectifier set.

Philips rectifier sets use mercury rectifiers which when started have a very low internal voltage drop. Apparently the rectifiers do not start conduction at an absolutely constant voltage, so that when the input used has a flat-topped wave form, such as a sinusoidal wave form, there is considerable jitter due to variation in the conduction period. With mains input, when the rectifier set is not working well, a record of the output indicates that several successive cycles have been missed altogether at times. When a peaked input, such as that obtained with a saturating inductor, is used, the current becomes a smoother function of the voltage drop across the rectifier, because a greater part of the controlling action is performed by external impedance. The improvement in the operation of the transformer rectifier set was great enough to warrant redesigning the circuit to provide peaked wave form under all conditions of operation.

3.38. The following circuit, in which the reactor is used as a shunt element, satisfies the above requirement and, in addition, avoids the two difficulties mentioned in section 3.36.

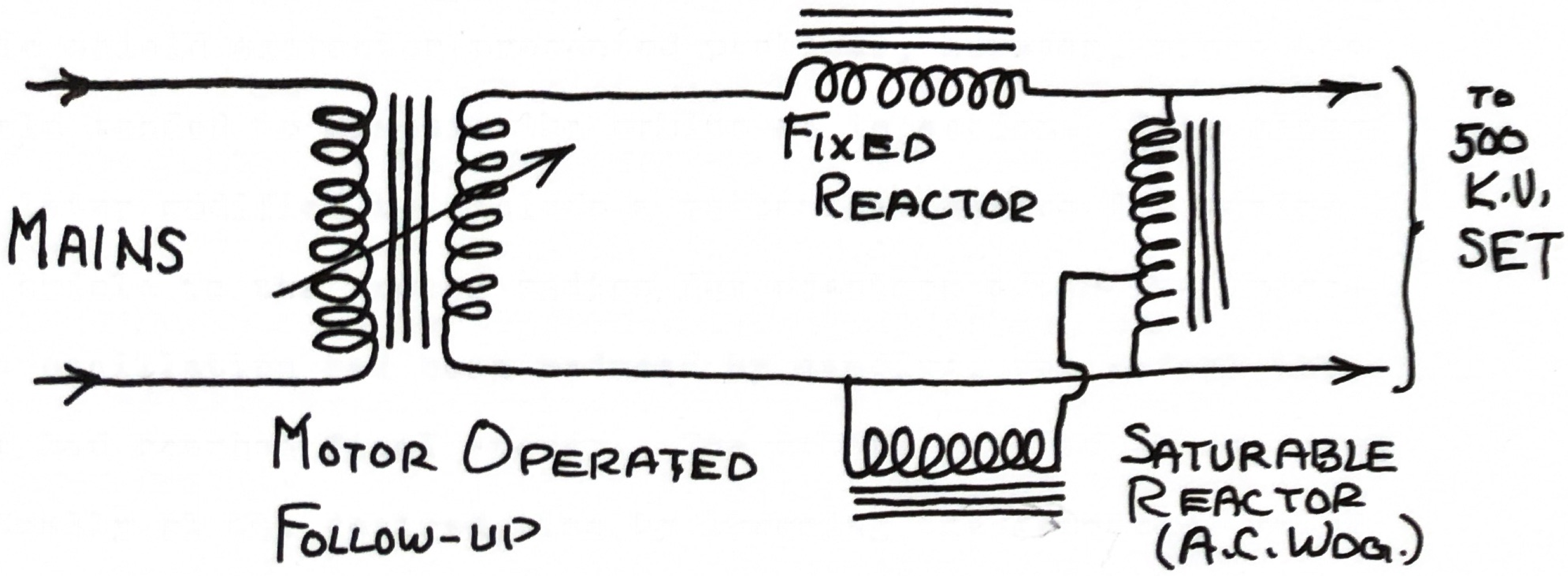


Fig. 3.38

The series reactor shown in figure 3.38 does not produce the type of instability mentioned in section 3.36 since this reactor does not have a D.C. winding and the core is therefore never biased. In the parallel (variable bias) reactor the effect is degenerative.

3.39. A year's experience with the circuit, during which the writer has received no complaints or calls for assistance from the operators, has shown the value of the simplicity of the apparatus. The expectation of smoother operation from the rectifier set because of the peaked wave form has also been realized. In addition, it has been found that the mains variations actually experienced can be handled adequately by the stabilizing system, in spite of the restriction on speed which was mentioned earlier.

CHAPTER IV

EXTRACTION

4.1. Some consideration has been given to a suitable method of extracting the particles after acceleration. It is shown that existing techniques are adequate for an extraction system capable of removing the beam in one revolution, but that gradual removal of the beam is not feasible.

4.2. The first successful method of extraction used in betatrons was the gradual expansion of the orbit into a magnetic shield at the end of the accelerating cycle.[16]

4.3. An electron beam has been extracted from the synchrotron at Harwell by the application of a short, high current pulse which temporarily weakens the field.

4.4. A system proposed for use on the Brookhaven synchrotron[17] followed the betatron scheme closely. The orbits were to be expanded by applying to electrodes an R.F. voltage of the frequency of the bbetatron oscillation, which would purposely excite betatron oscillation. The design of a magnetic shield extractor presented problems, however, since the shield tended to disturb the orbits at injection. This scheme was later modified to include a mechanical device for moving the shield to the proper radius for ejection after the injection oscillation had been reduced by damping, but before the ions had reached final energy. The orbits were to be expanded gradually at the desired time by lowering the frequency below its resonant value.

4.5. A high voltage pulse generator has been developed to deflect the beam of the 184 inch cyclotron in Berkeley, California.[18] The ions are deflected by a radial field which accelerates ions toward the centre of the cyclotron, the action resulting in a shift of the centre of rotation great enough to allow them to enter a magnetic deflector. The electrodes consist of two pairs of bars, 13 feet long, each pair raised to 100 K.V. with respect to ground in opposite directions.

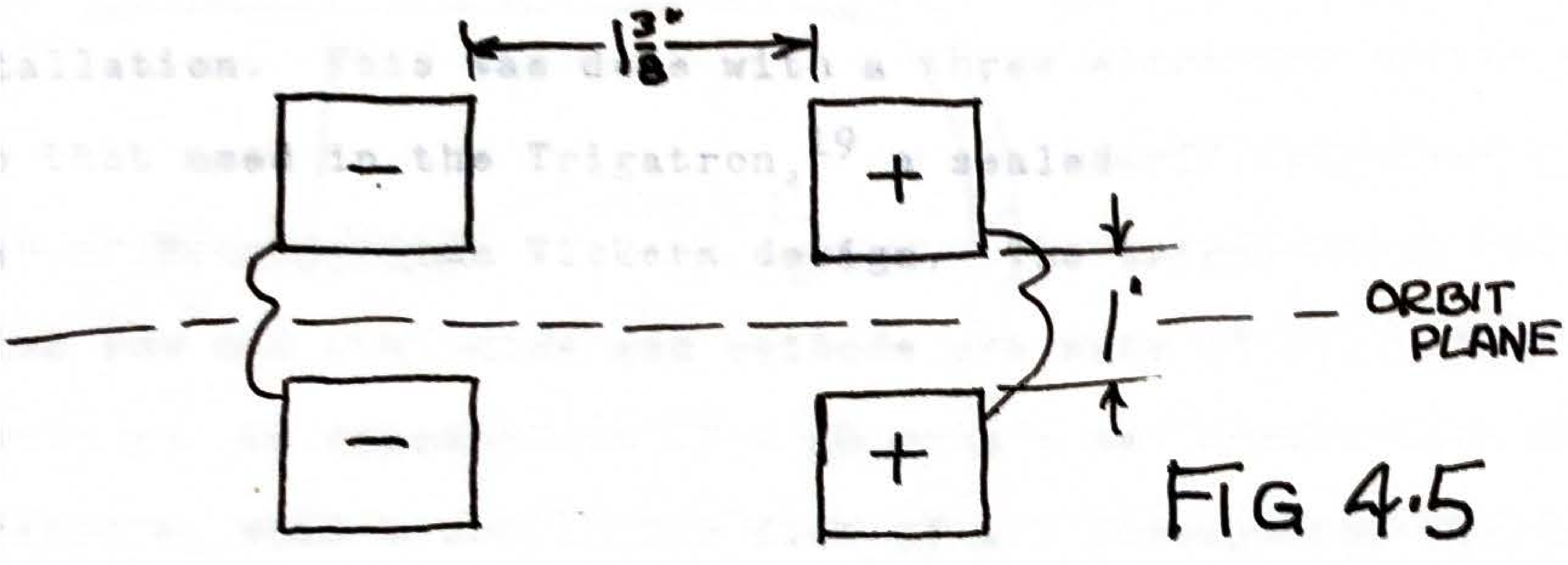


FIG 4.5

A highly specialized pulse transformer was developed to produce a difference of potential of 200 K.V. between the two pairs with a rise time of 0.15 microseconds. Each deflector represents a 600 micromicrofarad capacity which has to be charged up to 100,000 volts from ground through the transformer leakage inductance. Transformers with hypersil cores were designed in which the total inductance referred to the secondary is 7 microhenries. The voltage per turn is 6000. Approximately 90 per cent of the input energy is delivered to the cores as heat. The success of this transformer depends upon adequate water cooling, and upon purification of the insulating oil under vacuum and

its admission to the transformer case after the case has been thoroughly evacuated. The high gradient on the deflector bars has caused no vacuum insulation difficulties. This seems to be due largely to the short duration of the pulse, since such gradients with D.C. would certainly cause break-down.

4.6. In an experimental pulse extractor developed at Harwell, a voltage has been obtained comparable in magnitude and somewhat better in rise time than that of the Berkeley installation. This was done with a three electrode system similar to that used in the Trigatron,[19] a sealed-off triggered spark gap of Metropolitan Vickers design. The trigger is a thin tungsten rod and the anode and cathode are made of mild steel. The whole gap is operated at 20 - 40 pounds per square inch air pressure, with a continuous flow of air through the trigger hole in the anode. The flow is a few cubic feet per minute of dry air. A pulse of 30 to 50 K.V. is applied to the trigger from a 'Break modulator" circuit using a pulsed tetrode. The main gap voltage circuit is 120 to 180 K.V. and the repetition rate may be as high as 200 cycles per second.

4.7. Of the extraction systems described, the most attractive seems to be the one in use on the Berkeley cyclotron (section 4.5), since the deflection system is very simple and does not disturb the orbits in any way. The operation foreseen is roughly the following. When injection is desired, the frequency of the accelerating voltage will be locked, while the magnetic field continues to increase at its normal rate. The resonance value will thus rise slowly above the actual frequency,

introducing a progressive phase shift which causes the particles to acquire more than the normal value of energy per revolution. The orbits expand slowly. The deflector bars will be located at the edge of the stable field, the grounded pair being formed by an extension of the metal chamber wall. When the beam reaches

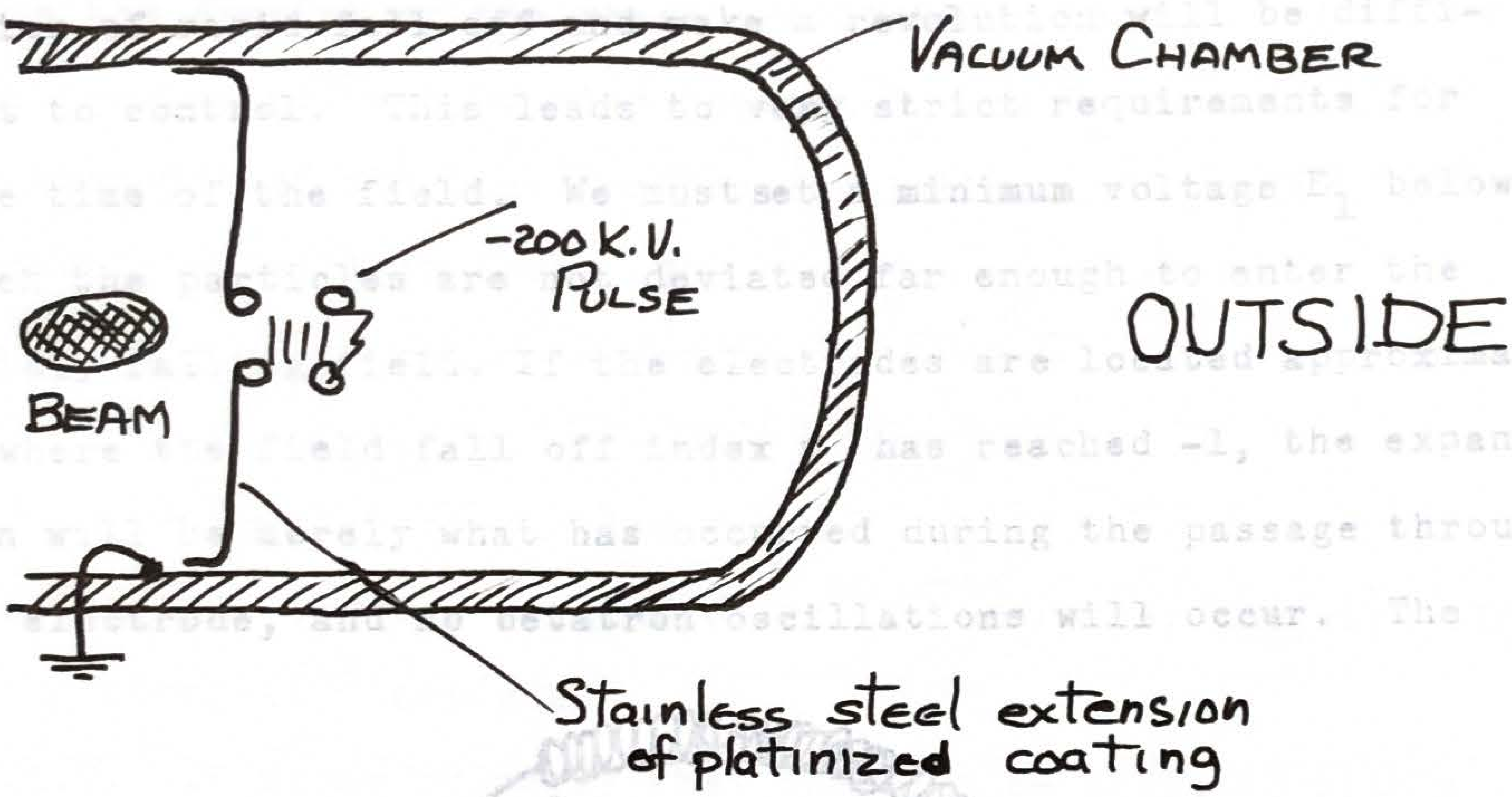


Fig 4.7

this point, a voltage of several hundred kilovolts will be applied to the bars causing a movement of about 1 inch in 12 feet of length. This movement is sufficient to bring the particles into the region where the field falls off very rapidly. Some place farther along the path, a stationary magnetic shield may be introduced, if necessary, without affecting the orbits during injection or acceleration.

4.8. The timing of the ejection pulse with respect to the movement of the particles is not difficult, since the expansion is very slow. This will be done by one of the methods described in Chapter V, the voltage rise being started when the beam reaches a suitable position. Because of the rapid field fall off, it seems necessary to have each particle subjected to the ejection field only once, since any particles which enter the region of rapid fall off and make a revolution will be difficult to control. This leads to very strict requirements for rise time of the field. We must set a minimum voltage E_1 below which the particles are not deviated far enough to enter the rapidly falling field. If the electrodes are located approximately where the field fall off index n has reached -1, the expansion will be merely what has occurred during the passage through the electrode, and no betatron oscillations will occur. The

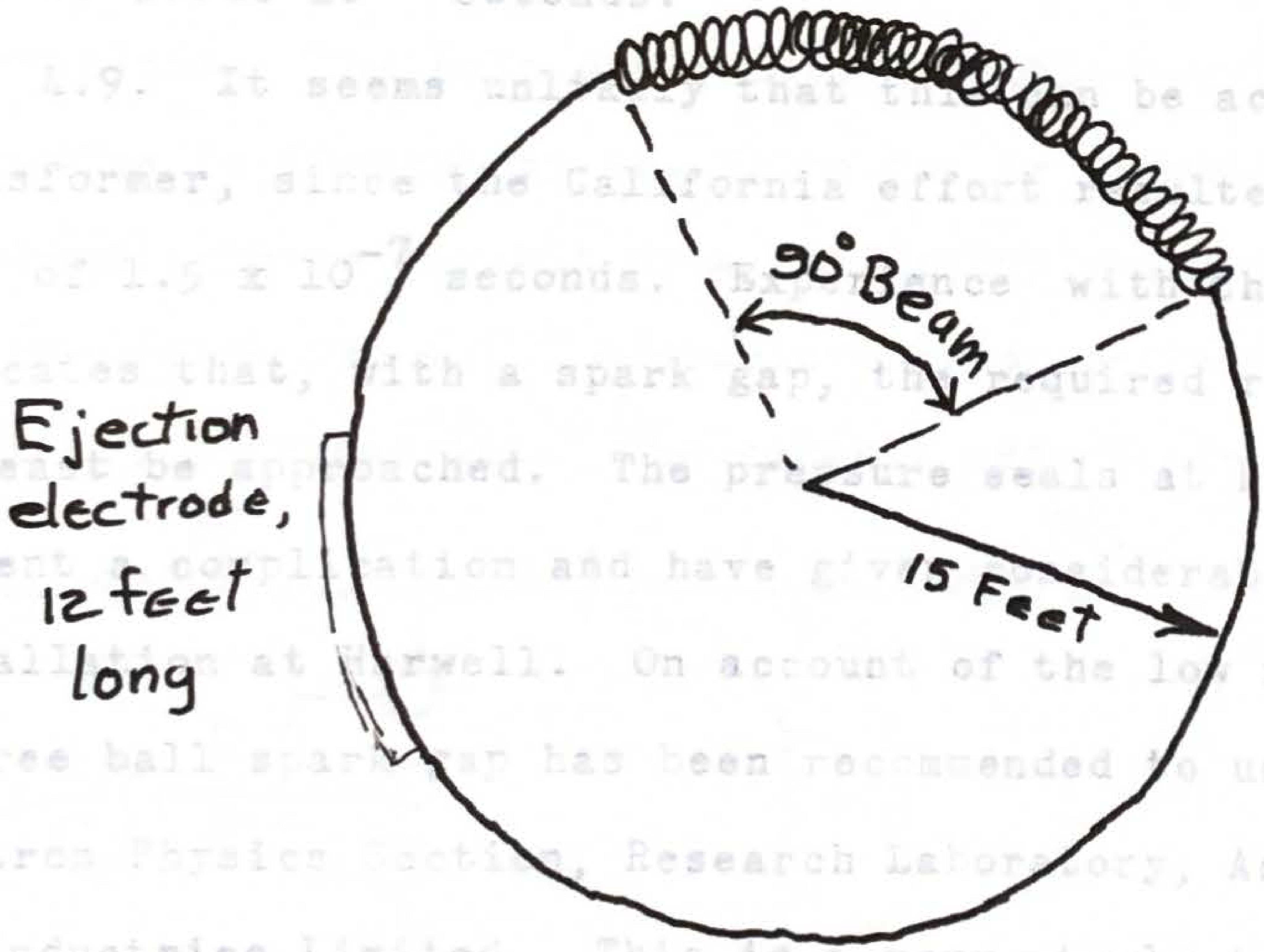


FIG 4.8

tolerance is then about that on the orbit position, or 0.1 inch. There is also some maximum voltage E_2 below which the particles will not be deflected enough to reach the exit hole after leaving the electrode. E_1 and E_2 would be of the order of 15 per cent and 85 per cent of the peak voltage. All particles which enter the electrode when the voltage, averaged over the passage, is between E_1 and E_2 will be lost. The expected length of the beam due to the damping of the phase oscillations is 90°. The time taken for the voltage to rise between E_1 and E_2 must then be less than 3/4 of the time for one revolution. Otherwise, all the particles will be lost. The fraction accepted is:

$$\text{Fraction accepted} = \frac{T_c - T_r}{T_c} \qquad (4.8.1)$$

T_c = length of the critical time

T_r = the actual rise time.

Since the time for one revolution is 10^{-7} seconds, the rise time must be about 10^{-8} seconds.

4.9. It seems unlikely that this can be achieved with a transformer, since the California effort resulted in a rise time of 1.5×10^{-7} seconds. Experience with the gap at Harwell indicates that, with a spark gap, the required rise time can at least be approached. The pressure seals at high voltage present a complication and have given considerable trouble in the installation at Harwell. On account of the low repetition rate, a three ball spark gap has been recommended to us by M.E. Haine, Electron Physics Section, Research Laboratory, Associated Electrical Industries Limited. This is a very simple type first described by E. G. White.[20] In the three ball gap, a standard

sphere gap has been broken up into two equal gaps in series by the introduction of a third sphere. By applying an impulse

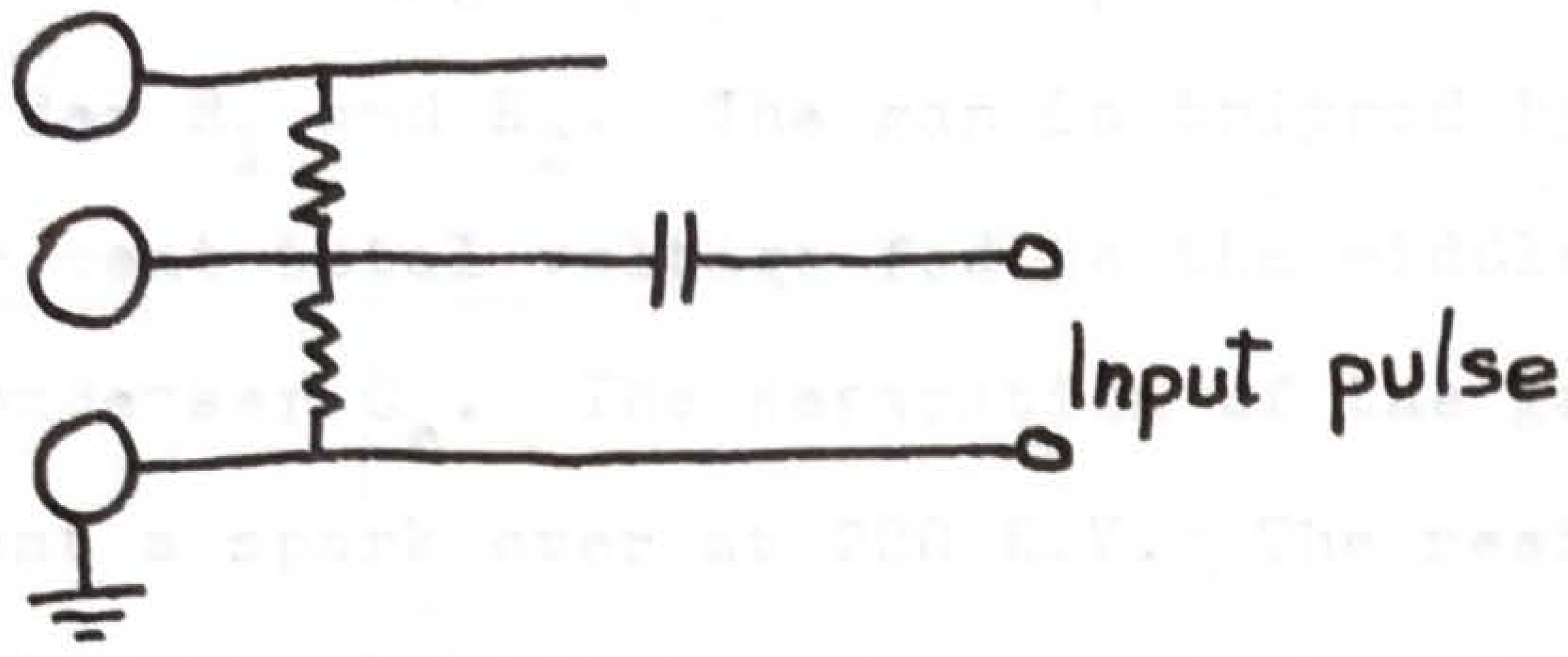


FIG 4.9.1

to the centre sphere when the gaps are charged nearly to the break-down value, break-down can be initiated. The delay in the initiation of the discharge is subject to considerable variation, but Mr. White thinks it is of the order of two to three microseconds. He did not use any extraneous irradiation in his experiments. The main difficulty in achieving the required rise time is thought by Mr. Haine to be reducing the inductance of the discharge circuit to a sufficiently low value. The maximum value for a 300 micromicrofarad deflector system is about 0.3 microhenries. The circuit recommended by Mr. Haine is:

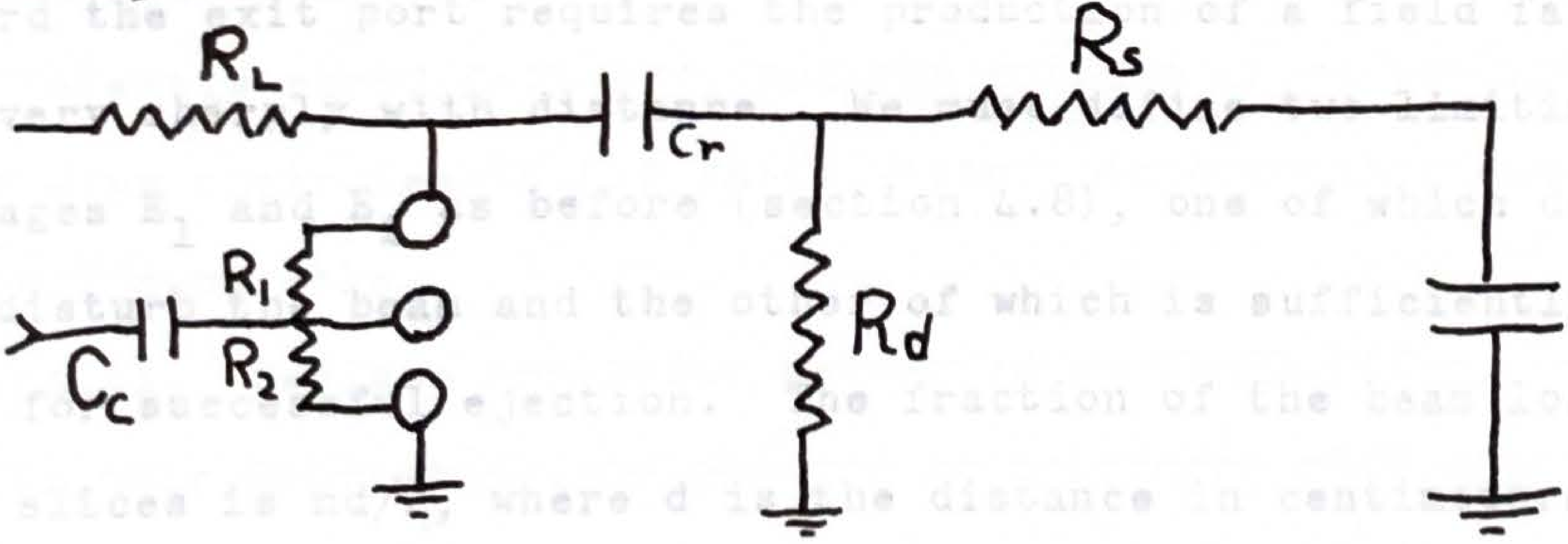


FIG. 4.9.2

Here C_p is the deflector system capacity; C_r is a reservoir condenser charged from a 200 K.V. source through the charging resistor R_c; R_d is the discharge resistance; G is a three ball gap, the centre ball being kept at half potential by a high resistance divider R_1 and R_2. The gap is tripped by a pulse of 10 to 20 per cent total voltage fed to the middle ball through the coupling condenser C_c. The separation of the gaps is just enough to prevent a spark over at 200 K.V. The resistance R_s is a damping resistance. A high speed sweep designed by Mr. K. McFayden for liquid dielectric break-down experiments is now in working order and should be capable of measuring rise times of the required shortness.

4.10. It has been suggested that it would be desirable to have the ejected pulse of protons spread out over at least a microsecond. The cross section of the beam in the Birmingham synchrotron has been estimated as roughly elliptical with vertical and horizontal axes of 1 cm. and 2 cm. respectively. This is due to scattering of the beam by the residual gas, without which the section of the beam would be of the order of a millimeter. To eject thin layers of the beam by attracting them selectively toward the exit port requires the production of a field falling off very sharply with distance. We must define two limiting voltages E_1 and E_2 as before (section 4.8), one of which does not disturb the beam and the other of which is sufficiently high for successful ejection. The fraction of the beam lost in n slices is nd/2, where d is the distance in centimeters

between the equipotential lines E_1 and E_2. For ten slices, half the particles in the beam will be lost when d = 0.1 cm.

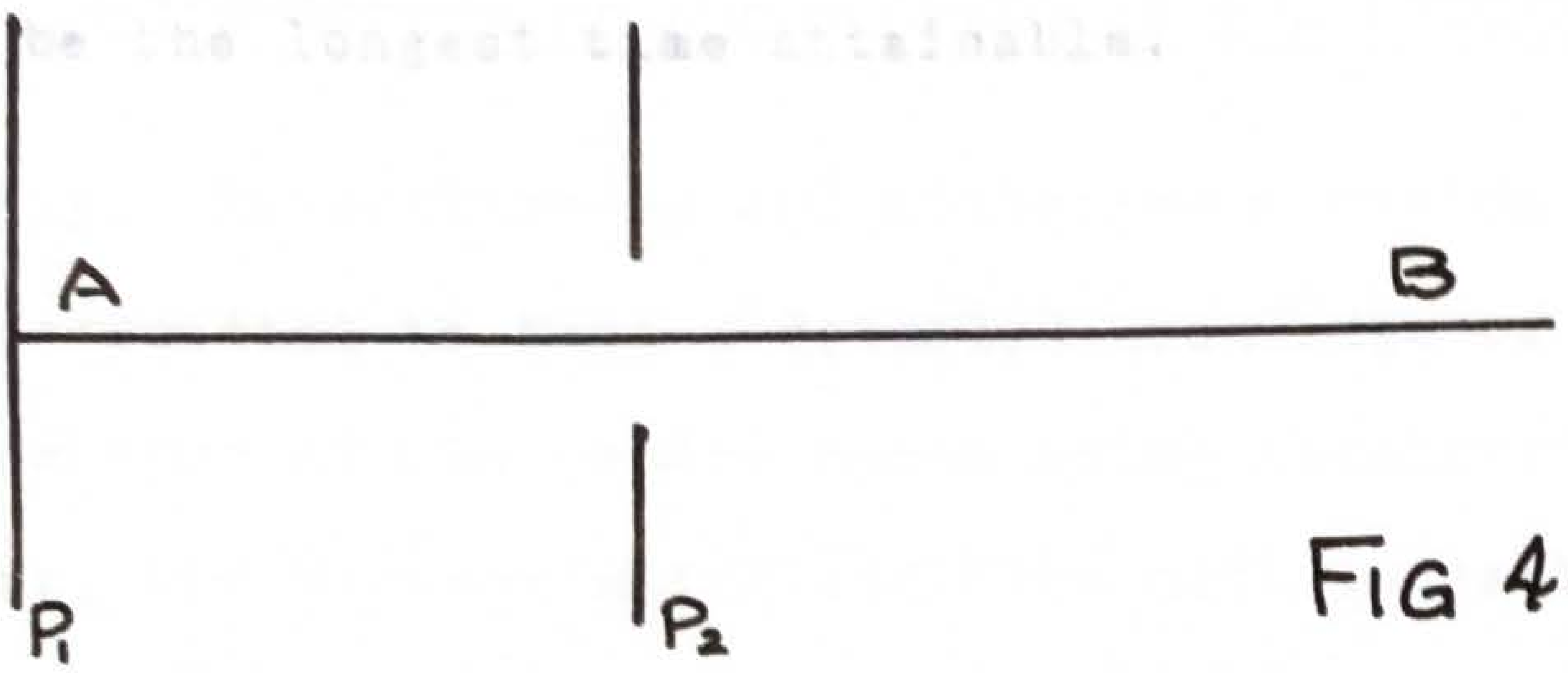


FIG 4.10

Consider two planes P_1 and P_2, one of which is pierced with a single slit at right angles to the plane of the paper, figure 4.10. The variation of field strength in the direction AB may be calculated from Schwarz's transformation.[21] If ℓ is the distance from the plane P_2 measured in units of the half-width of the slit,

$$\text{Field strength varies as } 1 + \frac{\ell}{\sqrt{\ell^2 + 1}} . \qquad (4.10.1)$$

To obtain the required rate of fall off, the diameter of the slit of figure 4.7 would have to be about 0.01 cm., that is, very much smaller than the vertical beam height. It is possible that a set of horizontal thin bars could be made to give a sufficiently rapid fall off, but the problem of supporting them over their length (12 feet) without intercepting the beam seems insuperable.

4.11. Devices are being developed at Chalk River, Canada, and in this laboratory, to measure coincidences with a resolving

time of the order of 10^{-9} seconds. It seems possible that the advantage of bringing the beam out gradually may not be great, at least if one assumes that a few microseconds (20 revolutions) would be the longest time attainable.

CHAPTER V

MEASUREMENT OF BUNCH SHAPE IN THE SYNCHROTRON

5.1. In setting up and operating a proton synchrotron, it is essential to have a detailed knowledge of the dimensions and position of the proton bunch being accelerated. In this chapter, the devices which would be most suitable for obtaining this information are discussed. The writer has made measurements on position finding electrodes of Brookhaven design, and has arrived at an improved form of electrode. These electrodes have been tested using bunches of electrons to simulate the proton beam. The writer has obtained circuits suitable for computing the beam coordinates throughout the accelerating cycle and presenting them as a function of time on a long persistence cathode ray tube. He has also devised a method of presenting beam phase and width throughout the cycle.

VACUUM CHAMBER AND ACCELERATING CYCLE IN THE BIRMINGHAM SYNCHROTRON

5.2. Before considering the devices which may be used to obtain information about the behaviour of the beam in a proton synchrotron, a brief review of the characteristics and sequence of operations in a proton synchrotron will be made from the point of view of possible monitoring equipment. Particles being accelerated in a proton synchrotron travel in a ring-shaped vacuum envelope placed in a magnetic field supplied by a ring shaped magnet (Figure 5.1). In the Birmingham machine,

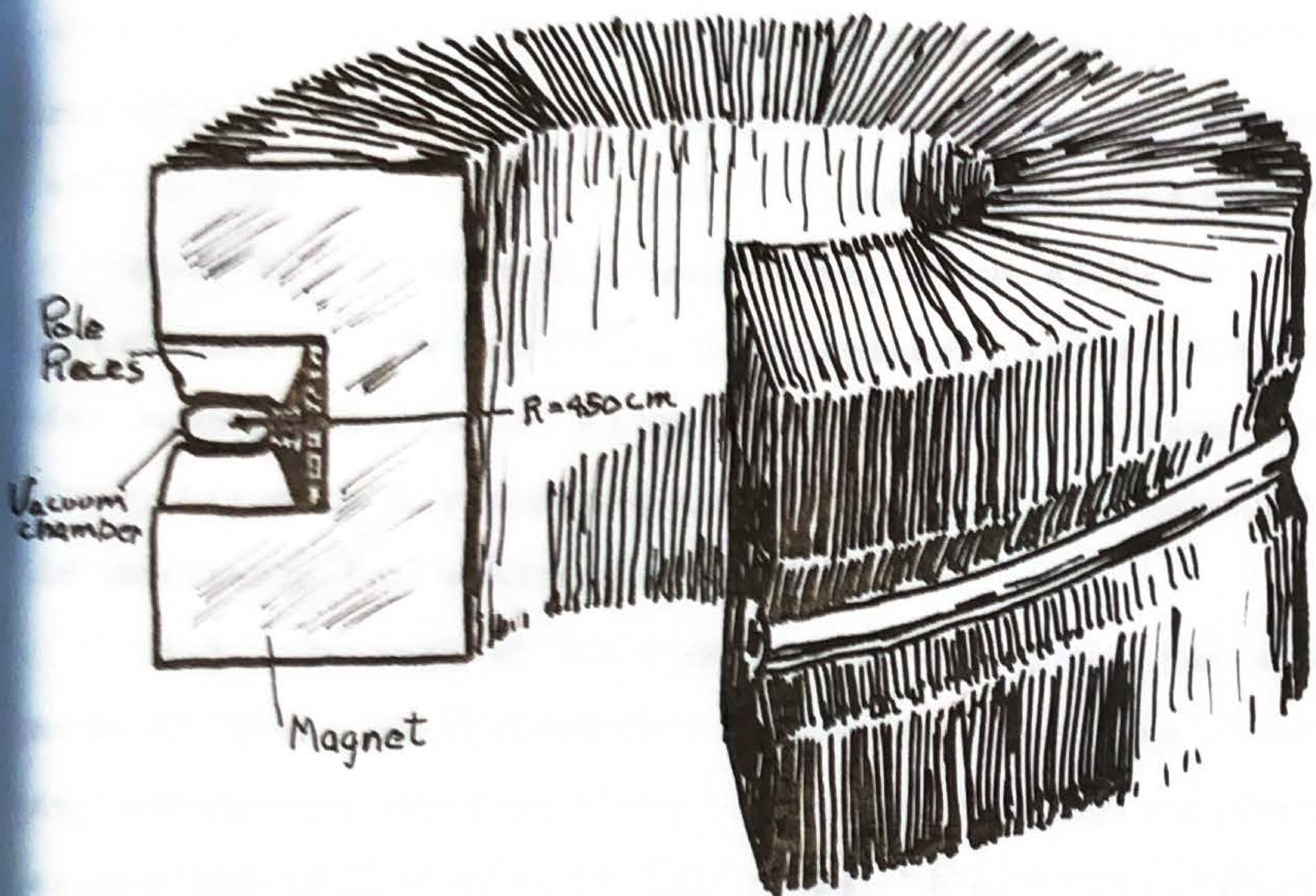


FIG. 5·1

the magnet consists of a ring made up of 2400 C-shaped iron pieces with the open side of the C facing outward. At the top and bottom of the gap so formed is an upper and lower ring of stampings forming the pole pieces of the magnet. Between these, its outer face accessible, the vacuum chamber is built up from cylindrical pieces with porcelain walls. These are about 46 cms. long, roughly rectangular in section, and provide a beam aperture 36 x 12 cms. The stable orbit is a circle of 450 cms. radius. The accelerating electrode occupies 105 degrees of this circle. Four or five of the 60 vacuum chamber sections are occupied by the injection apparatus, about eight by the ejection electrode. It is possible, if necessary, to include electrodes for beam monitoring in either injection or ejection sections. Thus, between half and two thirds of the space around the orbit is available for monitoring devices.

5.3. Because of the expense of providing beam aperture, none of these electrodes may reduce the available aperture. Any electrodes used on three sides of the chamber must be less than a few millimeters in thickness. Where thick electrodes are required, a special porcelain vacuum section may be used which provides extra space on the side of the vacuum chamber most remote from the magnet.

5.4. In order to prevent deflection of the protons by stray charges which might be picked up on an insulating wall, the inside of the chamber has been made conducting. An upper limit to the conductivity is set by the requirement that the

eddy currents induced by the changing field must not disturb the field distribution. The conducting coating consists of a lead glaze over the porcelain, of resistance a few megohms per square, overlaid with a thin layer of platinum, resistance a few ohms per square. Sections of the platinized coat may be isolated, if desired, by masking with tape during the platinizing. The lead glaze must be left to prevent the accumulation of charge on the parts not platinized. This procedure provides a simple method of making electrodes of practically any shape.

5.5. Two departures from ideal isolated electrodes are caused by the high resistance of the electrode itself, and the low resistance of the "insulation". The first depends upon the upper limit of frequency. When this is of the order of 10 megacycles, the first effect is not appreciable, unless the contact to the isolated electrode is poorly made. The second depends upon the lower limit of frequency and upon the size of the electrode, since the resistance to earth is proportional to the perimeter, and the capacity to the area. When the electrode extends all the way across the chamber, and when its shape is not too narrow in the direction of beam motion, a clearance of about half an inch between the electrode and the remaining earthed, platinized coat is satisfactory for operation at 300 K.C.

5.6. The magnetic field starts to rise when the magnet is switched across a 1,100 volt generator. When the field has reached 220 gauss, a signal from a proton resonance field measuring device starts injection of protons from a 500 K.V. D.C. accelerator tube. The protons are injected at the outside

of the operating range of orbits in the median plane. Injection lasts a few hundred microseconds. The protons are first spread out across the chamber, and spaced uniformly around it. For rising energy injection, they extend all the way across the chamber; for fixed energy injection, the limiting particles perform radial oscillations from one side of the chamber to the other. When the accelerating voltage is switched on, phase oscillations begin, and the protons outside the stable range of phases are accelerated to the inner wall. The protons now form a bunch somewhat greater than 180 degrees long, having all possible phases. The period of revolution of the protons at injection corresponds to an accelerating frequency of 290 K.C. This frequency rises at the rate of 25 megacycles per second up to about 5 megacycles, then the rate of rise steadily decreases. At the end of the accelerating period, 1 second, the frequency is 9.7 megacycles. The radial width of the beam shrinks fairly rapidly after injection to the value set by gas scattering (a few centimeters). The length of the bunch shrinks more slowly to about 90 degrees. The maximum possible number of particles accepted lies between 10^{10} and 10^{11}, of which at least half may be lost by gas scattering in the early stages of acceleration.

GENERAL DISCUSSION OF MONITORING DEVICES

5.7. Assuming that a good vacuum has been attained, a particle can be lost only if it strikes the walls. All problems of synchrotron operation involve the position of the particles

from the time they leave the ion source until they are put to use at the end of the cycle. The mechanism through which particles are most likely to hit the walls and be lost after injection is the phase oscillation, so a direct measurement of phase would be desirable as well.

5.8. When all possible care has been taken to measure by direct measurement the mechanical accuracy of magnet parts, vacuum chamber and injection apparatus, tests in which the positions of the particles are measured throughout the cycle may begin. Two stages are foreseen. In one stage the operating efficiency expected will be extremely low and the position finder will be chosen for its sensitivity. In the latter stage, a reasonable number of particles will be accelerated for at least a part of the cycle, and the detector will be chosen for convenience, accuracy, and amount of information supplied. Measurements of phase and the damping of phase oscillations will now be of value.

PARTICLE LOCATION IN THE EARLY STAGES OF SYNCHROTRON OPERATION

5.9. There is no doubt that the most valuable tool in the early stages of synchrotron operation will be the scintillation detector. It is, of course, possible to detect single protons when optical systems of moderate efficiency are used to connect a scintillation screen to a small commercial photomultiplier such as the RCA 931. Because of the distances involved, a somewhat lower sensitivity is obtained when this device is used in the synchrotron.

5.10. In the experiments on the model proton synchrotron at Berkeley, fourteen vacuum locks were provided at different points in the straight sections of the race track vacuum chamber. A $1\frac{1}{4}$ inch diameter lucite rod inside a copper tube was let in when desired on a "chevron" (post Wilson) sliding seal.

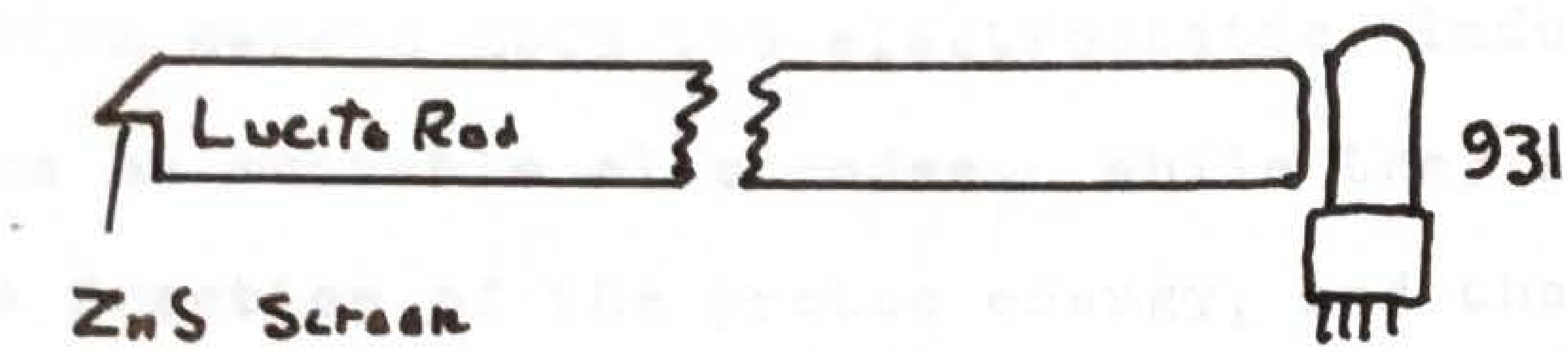


Figure 5.10

In the initial stages of operation the beam was tracked from the first turn out, step by step, by moving the detector a suitable distance into the chamber. When, after the first few revolutions, it was desired to detect the beam, the oscillator was stopped at the proper point in the acceleration cycle. The scintillation screen was located just outside the operating range, in an unobstructed detection range on the inside (that is, where the radius is smaller). A cathode ray tube swept with a time sweep showed the stopping of the oscillator, and the subsequent impact of the beam on the scintillation screen. The distance between the two events gave the radial position of the beam in the chamber just before stopping the oscillator.

5.11. It is, of course, essential to have the screen on the inside of the chamber. In the Birmingham proton synchrotron a small screen carried on a wire loop moved through a large

port hole in the vacuum chamber might be used. A simple optical system would connect with the lucite rod and photomultiplier. A set of movable stops for locating the edge of the proton beam would complete the equipment.

PARTICLE LOCATION AT LATER STAGES

5.12. The devices considered for the later stage of operation depend upon the electrostatic induction of the protons on suitable electrodes. While they absorb only a minute fraction of the proton energy, and therefore require more particles to operate them, they have the advantage of not interrupting the operation of the machine. This property is, of course, essential when use of the proton beam is contemplated. Even during experimental operation, devices possessing this property have the advantage of collecting a great deal of data and presenting it in an easily readable form. The minimum time between cycles is 10 seconds and the wear on the equipment per cycle is appreciable. Thus adjustments, to be effective, will have to be based on all the information available, in order to keep within reason the time and expense of reaching a desired operating condition. In addition, some information such as beam phase and width cannot be obtained from a reasonable number of observations with scintillation devices. In the following sections are described devices for measuring beam position, beam phase and width, beam intensity and current to the walls.

MEASUREMENTS OF BEAM POSITION BY DIFFERENTIAL INDUCTION

5.13. Consider a three dimensional rectangular coordinate system with the z-axis tangent to the stable orbit and the y-axis parallel to the magnetic field. It is desired to measure the x and y coordinate of the beam. The measurement of beam position by differential induction on electrodes was suggested by J. S. Gooden in 1946. Any two electrodes A and B will yield

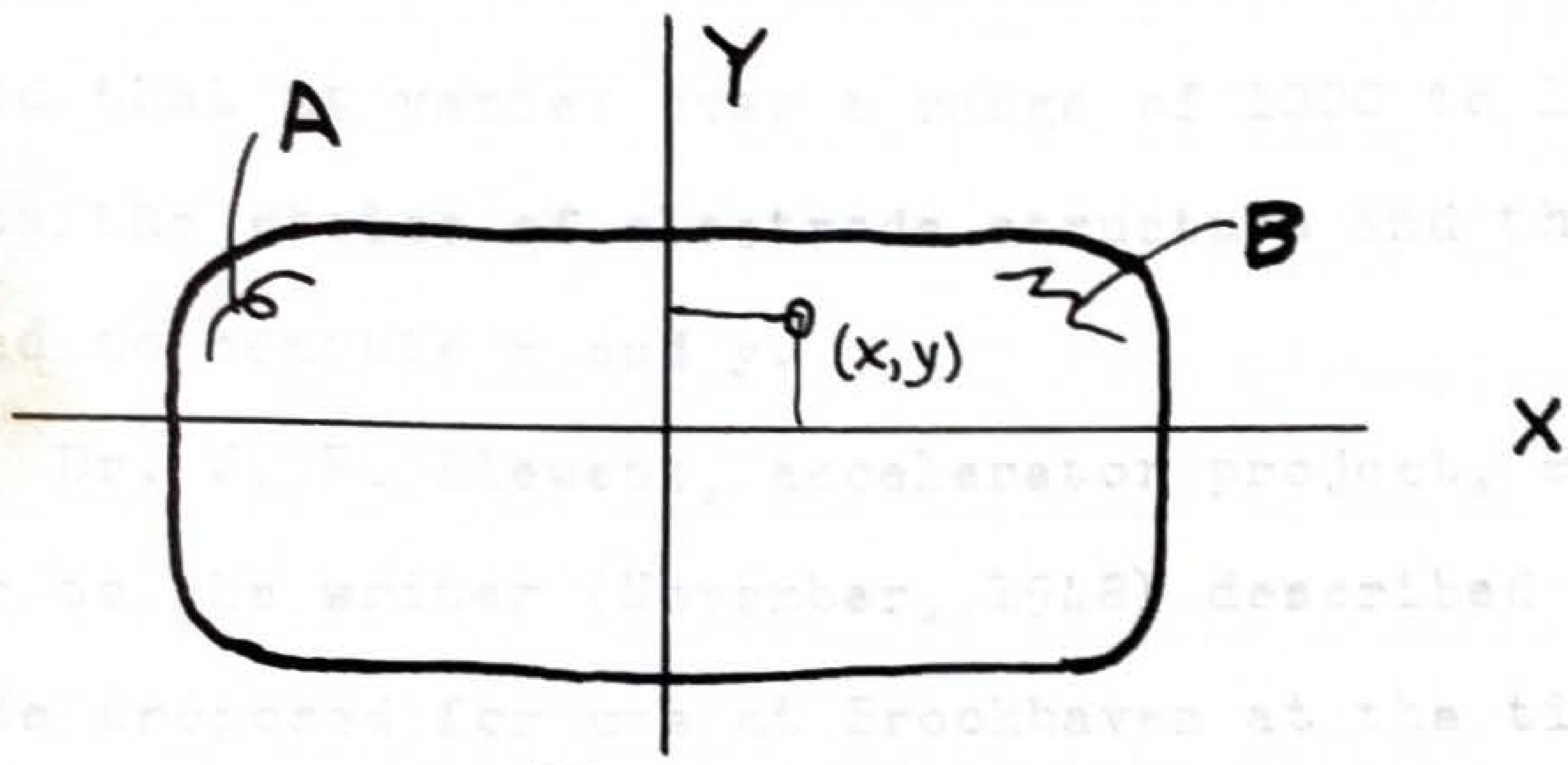


Figure 5.13

two voltages E_A and E_B which vary with beam position and intensity I:

$$\begin{aligned} E_A &= F_A(x,y,I) \\ E_B &= F_B(x,y,I) \end{aligned} \tag{5.13.1}$$

or

$$\begin{aligned} x &= G_1(E_A,E_B,I) \\ y &= G_2(E_A,E_B,I) \ . \end{aligned} \tag{5.13.2}$$

If these electrodes are to be suitable for position measurement, they must provide a single solution for x and y everywhere in the chamber (when I is known). If the accuracy to which the voltages must be measured is to be reasonable, $\frac{\partial x}{\partial E_A} + \frac{\partial x}{\partial E_B}$ and $\frac{\partial y}{\partial E_A} + \frac{\partial y}{\partial E_B}$ must lie below a reasonable bound everywhere in the chamber.

DESIRED ELECTRODE CHARACTERISTICS

5.14. It would be very desirable to find electrodes such that each voltage is a function of one coordinate only. This eliminates the need for the solution of simultaneous equations. For ease of presentation, and in order to obtain a uniform accuracy over the whole scale, it is desirable that position should be a linear function of the voltages picked up on the electrodes. It is to be noted that beam intensity is inevitably involved and that it varies over a range of 1000 to 1. This fact affects the choice of electrode structure and the form of circuit used to compute x and y.

5.15. Dr. J. P. Blewett, accelerator project, Brookhaven, in a letter to the writer (November, 1948) described the type of electrode proposed for use at Brookhaven at the time (figure 5.19). The writer started work from this point. The Brookhaven group also suggested the use of alternating voltages in measuring the characteristics of the electrodes.

MEASUREMENT OF ELECTRODE CHARACTERISTICS

5.16. By the characteristics of the electrodes is meant the nature of the functions E_A and E_B (5.13.1). Although the functions might be calculated by the methods of electrostatics, because of the shape of the electrodes considered, it is probably easier to measure the voltage on an actual electrode when a configuration of charges representing the beam have been placed inside the electrode. The earliest measurements made by the writer were made by introducing isolated charged

bodies into the space representing the chamber, and measuring the change of potential of the electrode under test with an electrometer. This can be done quite simply. The place of the beam may be taken by a small conductor having the shape of the beam, or by a number of conducting sections, insulated from each other, together having the shape of the beam. This structure is charged to a potential of 10 or 20 K.V. with an electrophorus. The conductor is brought up to the required position in the electrode structure and the change in potential due to the induced charge is noted. For a sphere of radius 1 cm. and an electrode to ground capacity of 1000 micromicrofarads, the voltage induced in the extreme position is about 10 volts, thus no great sensitivity is required of the measuring apparatus. Stray charges on the insulating support are difficult to deal with and the measurements are more trouble than A.C. measurements in other ways as well. However, D.C. measurements may be made on a system which represents the beam exactly, while the A.C. system necessarily involves an approximation.

5.17. The method used by the Brookhaven group for electrode characteristic measurement was to measure the capacity between the electrodes and a wire parallel to the axis. The capacity between wire and electrode was assumed to be proportional to the induction which would result from the beam. However, this is true only when the total capacity of the wire to all surfaces is constant. This may be seen by noting that the capacity of

the wire to the electrode approaches infinity as the distance between the wire and the electrode approaches zero. An exact analogy for the case of D.C. measurements (5.16), where the beam is represented by a distribution of fixed charges, would be an insulating rod with a layer of charge on it. If this rod were to approach the electrode, the potential would approach that reached by placing the charges on the electrode. This is the correct limit.

5.18. No exact analogue which allows the convenience of an A.C. measurement appears to exist. An improvement over the direct capacity measurement of section 5.17 results from using a wire fed from a constant current generator, and measuring the voltage induced on the electrode. As long as the capacity between a section of the wire and the conducting surfaces is the same over the whole wire, the charge distribution over the length of the wire remains uniform and the analogy is exact. The limit of induced voltage as wire approaches electrode may be seen to be finite, since, as the capacity increases, the voltage of the wire falls in the same ratio. To approximate a constant current source, a constant voltage generator with a finite series impedance is used. It is useful to note that the error is reduced by the reciprocal of the drop in output obtained upon inserting the series element.

THE BROOKHAVEN ELECTRODES

5.19. The Brookhaven electrodes consist of two pie-shaped electrodes with the points nearly touching. The electrode surfaces are all close to the chamber walls. The chamber walls

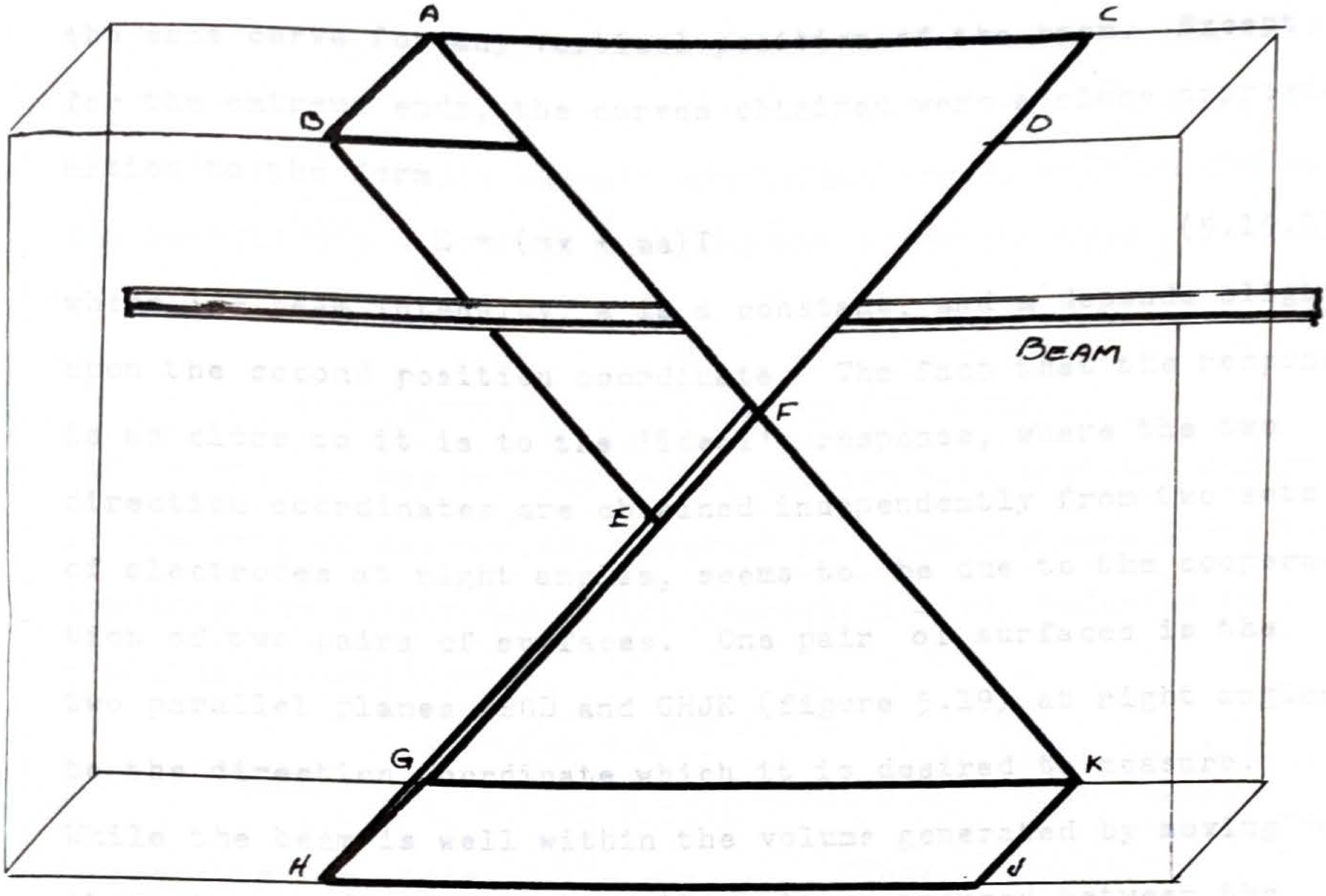


Figure 5.19

are at ground potential. Using the method of measurement described in section 5.18, the Brookhaven electrodes were found to give a characteristic

$$E_A = F_A(x,y,I)$$

which was essentially linear with x, except at the small end of the electrode. An ideal characteristic would be represented by a straight line which does not depend upon the second coordinate, y (section 5.14). That is, the electrodes used for measuring beam position in the horizontal direction should give

the same curve for any vertical position of the beam. Except for the extreme ends, the curves obtained were a close approximation to the form

$$E = (mx + ma)I \tag{5.19.1}$$

where I = beam intensity, a is a constant, and m depends slightly upon the second position coordinate. The fact that the response is as close as it is to the "ideal" response, where the two direction coordinates are obtained independently from two sets of electrodes at right angles, seems to be due to the cooperation of two pairs of surfaces. One pair of surfaces is the two parallel planes ABCD and GHJK (figure 5.19) at right angles to the direction coordinate which it is desired to measure. While the beam is well within the volume generated by moving these two surfaces together, the voltage induced between the two parallel plates is closely linear with distance. The other pair is the two double triangular plates BDEHJ and ACFGK. While the beam is close to either, the projected length of the beam on the closest plate is the part of the beam producing induction. This induction varies in a linear manner with distance also.

DESIGN CHANGE SUGGESTED BY MEASUREMENTS

5.20. Because of the departure of the curve from linearity at the ends, the design was changed to two sets of plates of the form ABCDEF, each extending all the way across the chamber. It will be shown that this change simplifies the computation

of position from electrode voltages. The two sets of plates may be adjacent, since, with essentially linear characteristics, capacitive cross-talk between electrodes serves only to decrease the sensitivity slightly, and does not alter the type of computation required.

COMPUTATION OF THE POSITION COORDINATES

5.21. It was pointed out in section 5.14 that the intensity of the beam is necessarily involved in the distance indication. Consider two electrodes having characteristics, referred to the chamber coordinates,

$$\begin{aligned} E_A &= (mx + m\tfrac{a}{2})\,I \\ E_B &= (-mx + m\tfrac{a}{2})\,I \end{aligned} \qquad (5.21.1)$$

where a is the width of the chamber. Since

$$\frac{E_A - E_B}{I} = 2mx \quad , \qquad (5.21.2)$$

this ratio would be proportional to x if m were constant. However, m actually depends slightly upon y. We therefore consider the ratio $\frac{E_A - E_B}{E_A + E_B}$ which is also proportional to x. We have

$$\frac{E_A - E_B}{E_A + E_B} = \frac{2mxI}{maI} = \frac{2x}{a} \; . \qquad (5.21.3)$$

Thus the dependence of m on y does not affect the ratio $\frac{E_A - E_B}{E_A + E_B}$. This has the additional advantage that it is not necessary to measure I separately. A similar apparatus is suitable for the measurement of the y coordinate. The discussion of the mechanism of operation given in section 5.19 still applies, although now the pair of parallel planes ABCD, GHJK determines the characteristic over a greater range of beam positions. The measured characteristics are shown

in figure 5.26.1.

5.22. If two mechanically similar electrodes are used for the signals E_A and E_B, the centre indication is correct no matter what may be the departure from the assumed relation

$$y = (mx + ma)I$$

where a is a constant. The circuit which computes the distance coordinate is also designed to provide the highest accuracy at the centre where the beam is most likely to be. The errors, except at the walls, will be less than those involved in the displaying of the final results.

FURTHER TESTS OF POSITION FINDING SYSTEM

5.23. If they are to be of value in the early stages of synchrotron operation, the position finding devices should be as thoroughly tested as possible before they are actually used on the synchrotron. On this account, it was felt desirable to test the electrodes using an electron beam which would approximate the characteristics of the proton beam as closely as possible. A beam of electrons consisting of bunches travelling at the same speed as the protons at a chosen point in the acclera-tion cycle, and having the same length as the proton beam, was made to pass through the electrodes at different positions, while the voltage induced on the electrodes was measured. Although the experiment involves only well-known techniques, the difficulties of producing the various vacua required are considerable. The writer has relied entirely upon vacuum

apparatus and techniques developed by the synchrotron vacuum group. He is very grateful for the help given him by Dr. L. Riddiford, without whose assistance it would have been impossible to carry out the experiment.

5.24. A proton beam having a constant number of charges per unit length induces, in a Faraday cage which it travels through but does not touch, a peak voltage which is independent of the proton velocity, since the number of charges inside remains a constant. The current represented by the beam increases as the proton velocity. At 1 Bev., the peak current represented by 10^8 charges in the Birmingham proton synchrotron is about 300 microamperes. An electron gun delivering electrons at the same velocity will have to supply 300 microamperes to give the same peak voltage. The gun voltage in this case is 500 K.V. For protons at 3.7 Mev., $\frac{v}{c}$ is 0.1, the synchrotron oscillator frequency (stable orbit radius 450 cm.) is 1.06 megacycles, and the electron energy for the same speed is about 2 K.V. The gun current to give the same peak voltage as 10^8 protons is 34 microamperes. These were the operating conditions chosen. A gun current of several milliamperes was considered convenient for measurement.

5.25. In order to reduce the vacuum requirements to a minimum, and to allow frequent opening of the vacuum system, a gun with a tungsten cathode was used. This was considered feasible because the beam width was not required to be smaller than 1 cm. or so. The trumpet-shape accelerating electrode was patterned after the design finally adopted in the narrow

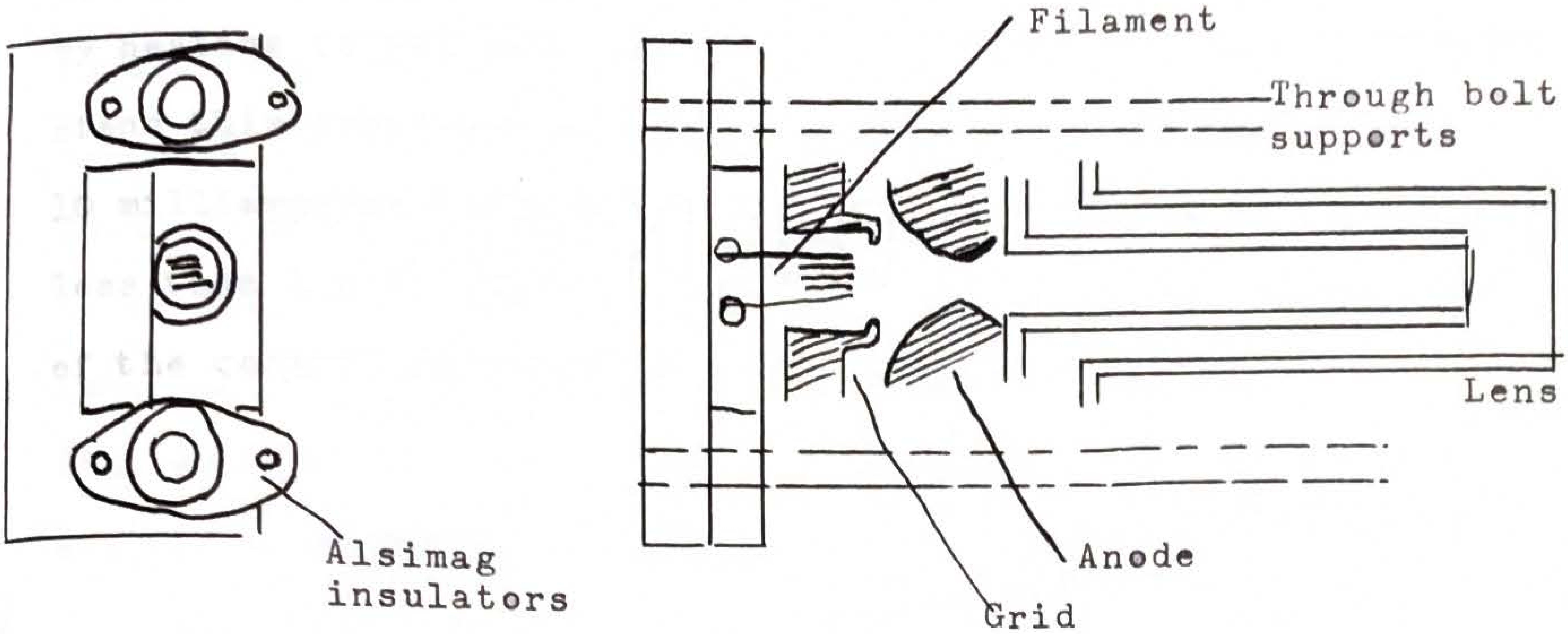


Figure 5.25.1

beam klystrons. It has been shown to be less affected by decentering of the cathode than a hole in a flat plate, and it was hoped it would be equally insensitive to the irregularities in the tungsten cathode. The cathode was a 5 turn helix of 0.5 mm. diameter tungsten wire about 5 mm. in outside diameter, and 5 mm. in length. It was operated about 2500° K where the emission is 260 milliamperes per square centimeter. The life, according to Parker,[22] is 45 hours at this temperature. Owing to breakages when taking the gun apart, no filament was ever run for this long, but at least there were no burn-outs. A simple device was used for winding the helix, and screw clamps were used to hold it in place. Because no activation was required, and because it was not subject to poisoning, the pure tungsten filament was very convenient. The most serious disadvantage is the heat produced. It was necessary to design the supports to withstand considerable heat. The assembly

was finally mounted on "alsimag" insulators. These were tested by heating to red heat and plunging in water. They appear to stand this treatment as well as quartz. It was possible to get 10 milliamperes through the 1/8 inch hole in the electrode with less than 1 milliampere current to the electrode. The effect of the control grid was as shown in figure 5.25.2.

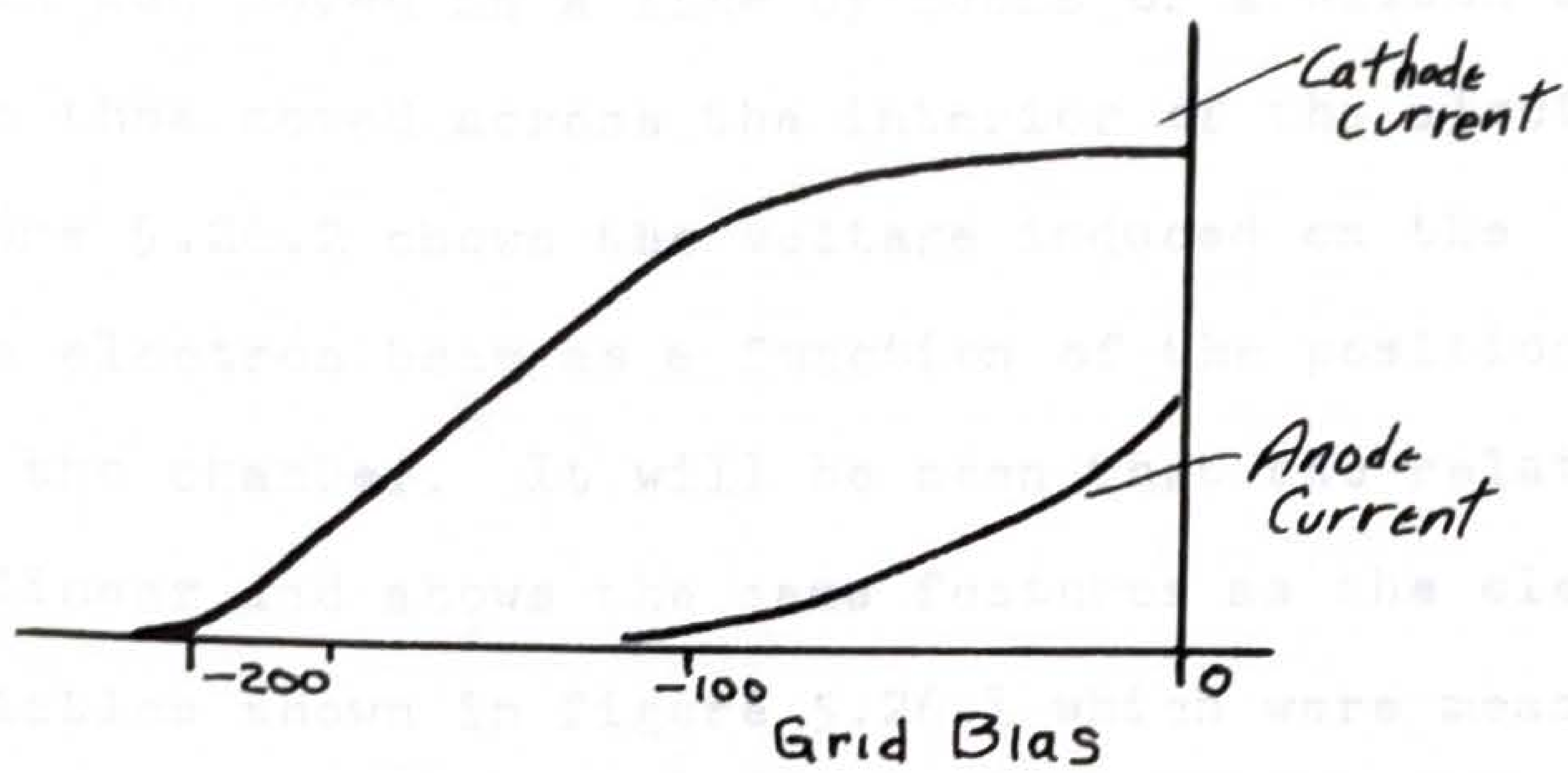


Figure 5.25.2

When the filament was moved back, the bias voltage for low anode current moved toward more positive grid voltages, as would be expected. The position chosen was that which gave low anode current at nearly zero grid volts. The front surface of the filament was between 3 and 4 millimeters behind the front surface of the grid at this point. Filament and grid were contained in a sub-assembly so that the position of the filament could be adjusted easily before attaching the other components. The details of the gun design are shown in Figure 5.25.1. With the operating grid voltage zero, the gun could be modulated conveniently by applying a few hundred volts of R.F. voltage to the grid. The initial tests were done in a small bell jar

sealed to a brass plate with Apiezon Q. The gun was then moved to a vacuum chamber 4 feet long which would house the $\frac{2}{3}$ scale model of the position finding electrodes. Here the beam was found to spread out in a cone about 7 degrees wide, and thus to be too wide at the top of the chamber. A long focal length lens was added, and the beam was focused on the end of the chamber. The gun was moved in a line by means of a Wilson seal, and the beam was thus moved across the interior of the electrode.

5.26. Figure 5.26.2 shows the voltage induced on the electrode by the electron beam as a function of the position of the beam across the chamber. It will be seen that the relation so obtained is linear and shows the same features as the electrode characteristics shown in figure 5.26.1 which were measured by the method of section 5.18. The characteristics in both cases consist of a long section with constant slope terminated at the apex end by a curved section. The characteristic measured using the electron beam presents one feature which is not found in the characteristic measured by the wire analogue. When the electron beam is moved close to the wall some of the electrons begin to be collected on the wall and thus have a greater effect on the variation of potential of the electrode than a normal electron which spends a comparatively small fraction of the cycle passing through the electrode. This results in a rise of the curve above the straight line for a short section close to the wall. The departure from the straight section is not noticeable until the beam is within

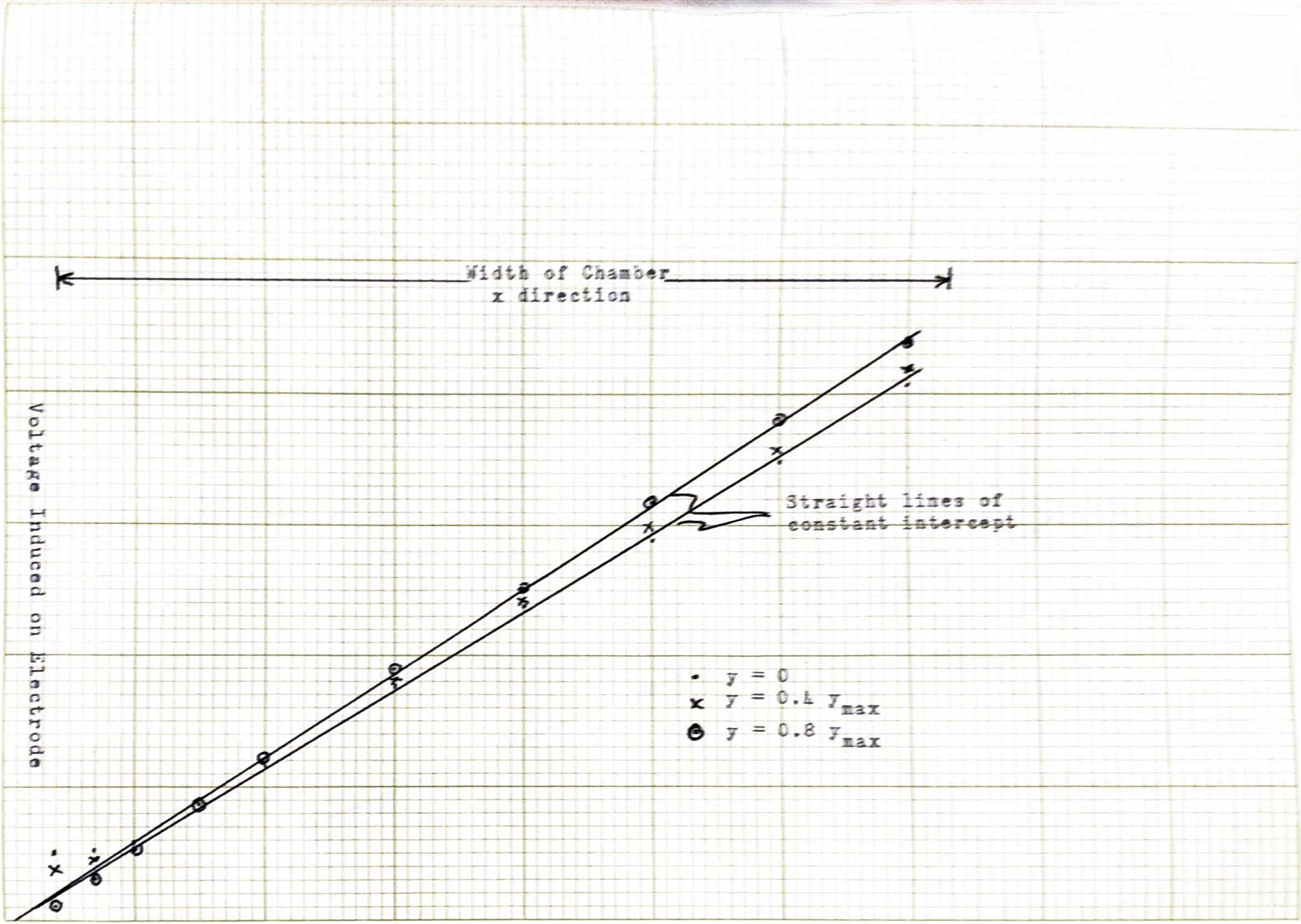

Width of Chamber
x direction
Voltage Induced on Electrode
Straight lines of
constant intercept
· y = 0
x y = 0.4 y max
⊙ y = 0.8 y max

FIGURE 5.26.1

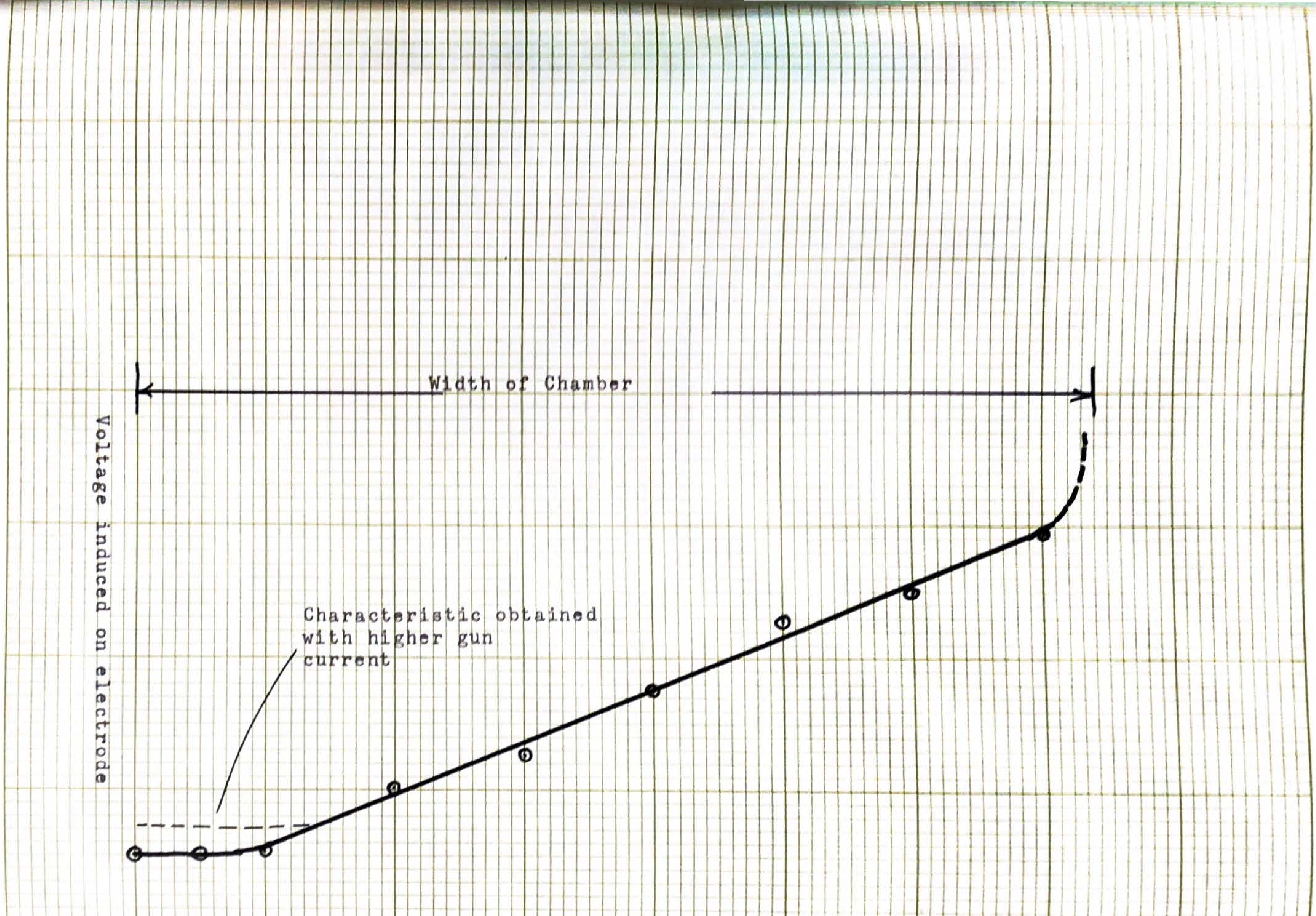

Width of Chamber
Voltage induced on electrode
Characteristic obtained with higher gun current

FIGURE 5.26.2

3 or 4 mm. of the wall. It would have been desirable to use a range of electron velocities. However, the design of the gun did not permit changing the voltage over a significant range.

5.27. Agreement has thus been obtained between the electrode characteristics measured with a beam of moving electrons having the same general features as the proton beam and the electrode characteristics measured using the charges induced on a wire to simulate the proton beam. The experimental arrangement in the case of the wire is of course much more convenient, and it is possible to define the position of the beam with greater accuracy. Some errors, such as slight defocusing of the beam at the change-over points in the modulating cycle and currents due to the ionization of residual gas, are avoided in the wire system of electrode characteristic measurement. It is clear, however, that the similarity between the moving electrons and the proton beam in the proton synchrotron is much closer than the similarity between the wire model and the proton beam. The tests with the electron beam thus serve to establish confidence in the results obtained by the wire model.

DISPLAY OF THE COORDINATES

5.28. Since the maximum amount of information must be available at the end of each cycle, the results will be presented in the form of a graph of x or y coordinate versus time, drawn on a long-persistence cathode ray tube. To draw this graph

from information supplied by the position electrodes according to the method of section 5.21, a voltage proportional to x

$$E_x \propto \frac{E_A - E_B}{E_A + E_B} \qquad (5.28.1)$$

is required.

COMPUTATION OF THE COORDINATES

5.29. It has been pointed out earlier that the intensity of the beam may not be assumed known, and may vary over a range of 1000 to 1. The range of variation of the electrode voltages is this ratio times the ratio of the greatest distance to the smallest distance to be distinguished. The total range is thus about 100,000 to 1. This range of variation appears in numerator and denominator of the expression giving beam position as a function of electrode voltages (5.21.3). The circuits required to convert electrode voltages into a voltage representing beam position must be capable of handling this wide range of intensities. Some of the methods used to compute a ratio depend upon addition of the outputs of devices with exponential characteristics. Elements suitable for some purposes are germanium crystals and selenium cells. An older element is the retarding-field diode, which has found a recent application in a wattmeter.

5.30. None of these elements is as stable as elements with linear characteristics. Further, none is capable of covering the desired range. The two circuits to be described depend upon the relations which obtain in an essentially linear

amplifier of adjustable gain:

$$E_o = E_i \cdot G \tag{5.30.1}$$

where E_o is the output voltage, E_i the input voltage, and G is the gain. This relation provides a method of multiplication. When a feedback loop is arranged to hold E_o constant,

$$E_i \cdot G = K$$

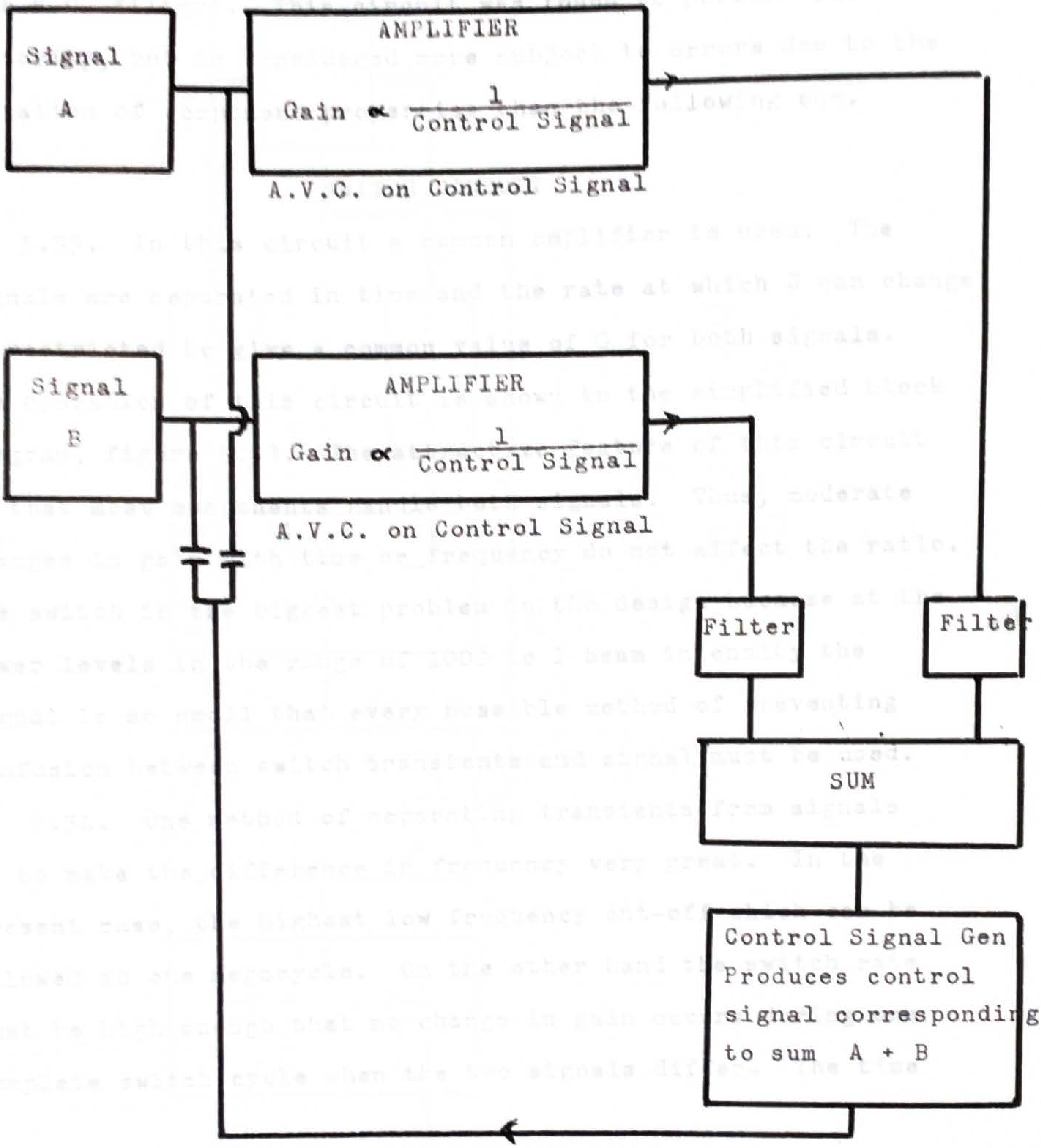
$$G = \frac{K}{E_i} \quad . \tag{5.30.2}$$

The method of division is thus provided.

CONTROL SIGNAL CIRCUIT

5.31. In this circuit, shown in block diagram form, figure 5.31, two separate amplifiers are used, and the quantity G is carried over from one to the other by a third signal. This signal is sufficiently different in frequency from the two voltages E_A and E_B that it can be separated easily from them. The main problem is the performance of the second feedback loop which involves the control signal. This is discussed in Appendix B .

5.32. Although the two amplifiers need not have a particularly flat gain versus frequency characteristic, this characteristic must be matched in both amplifiers. This is not too easy to ensure at the higher frequencies. A small change in the absolute gain is not important. Constant current coupling to the control signal generator may be used to compensate for changes in electrode or cable capacity, should they occur. The control signal generator is a conventional fixed amplitude oscillator feeding



CONTROL SIGNAL RATIO CIRCUIT

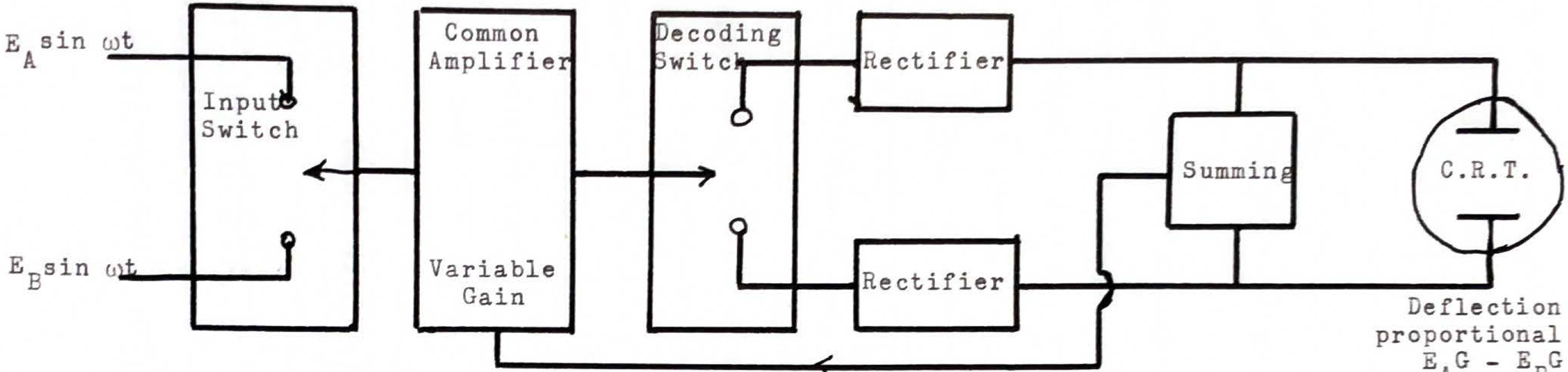
FIGURE 5.31

an AVC amplifier designed according to the principles discussed in Appendix C . The relation between control signal and amplifier output must be logarithmic if the loop gain is to remain constant. Satisfactory separation of the signals was achieved with R.C. filters. This circuit was found to perform satisfactorily, but is considered more subject to errors due to the variation of component properties than the following one.

SWITCH CIRCUIT

5.33. In this circuit a common amplifier is used. The signals are separated in time and the rate at which G can change is restricted to give a common value of G for both signals. The operation of this circuit is shown in the simplified block diagram, figure 5.33. The attractive feature of this circuit is that most components handle both signals. Thus, moderate changes in gain with time or frequency do not affect the ratio. The switch is the biggest problem in the design because at the lower levels in the range of 1000 to 1 beam intensity the signal is so small that every possible method of preventing confusion between switch transients and signal must be used.

5.34. One method of separating transients from signals is to make the difference in frequency very great. In the present case, the highest low frequency cut-off which can be allowed is one megacycle. On the other hand the switch rate must be high enough that no change in gain occurs during one complete switch cycle when the two signals differ. The time



Deflection proportional to

$$E_AG - E_BG$$

$$= G(E_A - E_B)$$

$$= \frac{E_A - E_B}{E_A + E_B} K$$

Gain Control
(Acts to keep sum constant)

$$E_AG + E_BG = K$$

$$G = \frac{K}{E_A + E_B}$$

SWITCH RATIO CIRCUIT

FIGURE 5.33

constant of the AVC filter must be fast enough to handle the fastest changes in beam intensity expected. The types of cut-off which may be used are limited by the Nyquist condition on the AVC loop. (Appendix E). The switch frequency was chosen at the highest value which allowed satisfactory separation of signals from switch transients after the other methods had been pushed as far as possible.

5.35. A second method of separating switch transients from signals is the use of the highest level possible at the switch. The limit here is set by the distortion in the switch and is fairly inflexible. It may be extended somewhat by the use of feedback (when the switch includes an amplifier). To reach the prescribed limit, the switch must be preceded by amplifiers shown on the block diagram (figure 5.33).

5.36. A third method of reducing switch interference is delaying the closure of the decoding switch until the transients have died out. This possibility may be taken into account by considering, in design, the length of the transient rather than its magnitude. This assumes that no change in gain is caused by the transient outside the time limit inside which it compares with the signal in magnitude. This condition can be fulfilled without introducing further limitations by proper design of grid circuits. This is discussed in Appendix C on AVC amplifiers.

5.37. In addition to the transient problem, the following requirements must be considered in the design. All parts must be linear to the accuracy required of the final position indication, that is, a few per cent. The degree of linearity required

is, of course, much less than if the two signals were handled separately. The range of intensities handled becomes smaller as one proceeds from the input toward the output, and at the output the range of intensities is only that of the ratios covered. This is important in considering such elements as the diode, which could not possibly be designed to be linear over a range of 100,000 to 1. The frequency response of the few parts of the system which handle one signal alone must be matched to within the permissible ratio error.

5.38. On the graph representing beam position versus time a line formed by feeding equal amounts of signal from an oscillator to the pickup electrode will be superimposed. The simplicity of this divider circuit will ensure that the line so formed is the line corresponding to a centered beam. It will be drawn immediately after the synchrotron accelerating cycle has been finished. For identification, this line will be made a broken line.

PHASE AND WIDTH OF THE BEAM

5.39. As acceleration proceeds, the beam length is reduced due to the damping of the phase oscillations of the particles about the equilibrium phase. Gooden, Jensen and Symonds [23] have shown that there are four methods for adjusting the behaviour of phase oscillation amplitude. These are: (a) changing the rate of change of R.F., (b) varying the voltage amplitude during acceleration, (c) varying the way in which the magnetic

field increases with time, (d) shaping the faces of the accelerating electrode. Since, however, sufficient damping is not easy to achieve, it is important to know exactly how the amplitude of oscillation varies with time. If a serious loss of particles should occur, a study of the behaviour of beam width prior to the loss would show whether or not this loss was caused by an increase in the amplitude of phase oscillations. In addition, the particles might move outside the stable range of phases, due to a sudden unexpected jump in the frequency of the accelerating R.F. A record of the phase of the beam referred to the R.F. would then be of value.

5.40. The data required can be gathered from a Faraday cylinder, short compared to the length of the beam, placed in the path of the beam. The induction in the cylinder is then proportional to the instantaneous density of the beam at that point, that is, to the number of particles inside. The wave form developed is displayed on a cathode ray tube, using the two cathode ray tube variables: spot intensity and x-position of the spot. The x-position of the spot corresponds to the 360 degrees rotation of the beam in the synchrotron. Thus in one complete rotation of the beam the spot on the display tube moves once from left to right tracing out a horizontal line (figure 5.40). While moving from left to right the spot is made to change in intensity to correspond with the density of the beam. This process is repeated for every revolution of the proton beam. During the acceleration cycle, however, the line is continuously moved downward, producing an

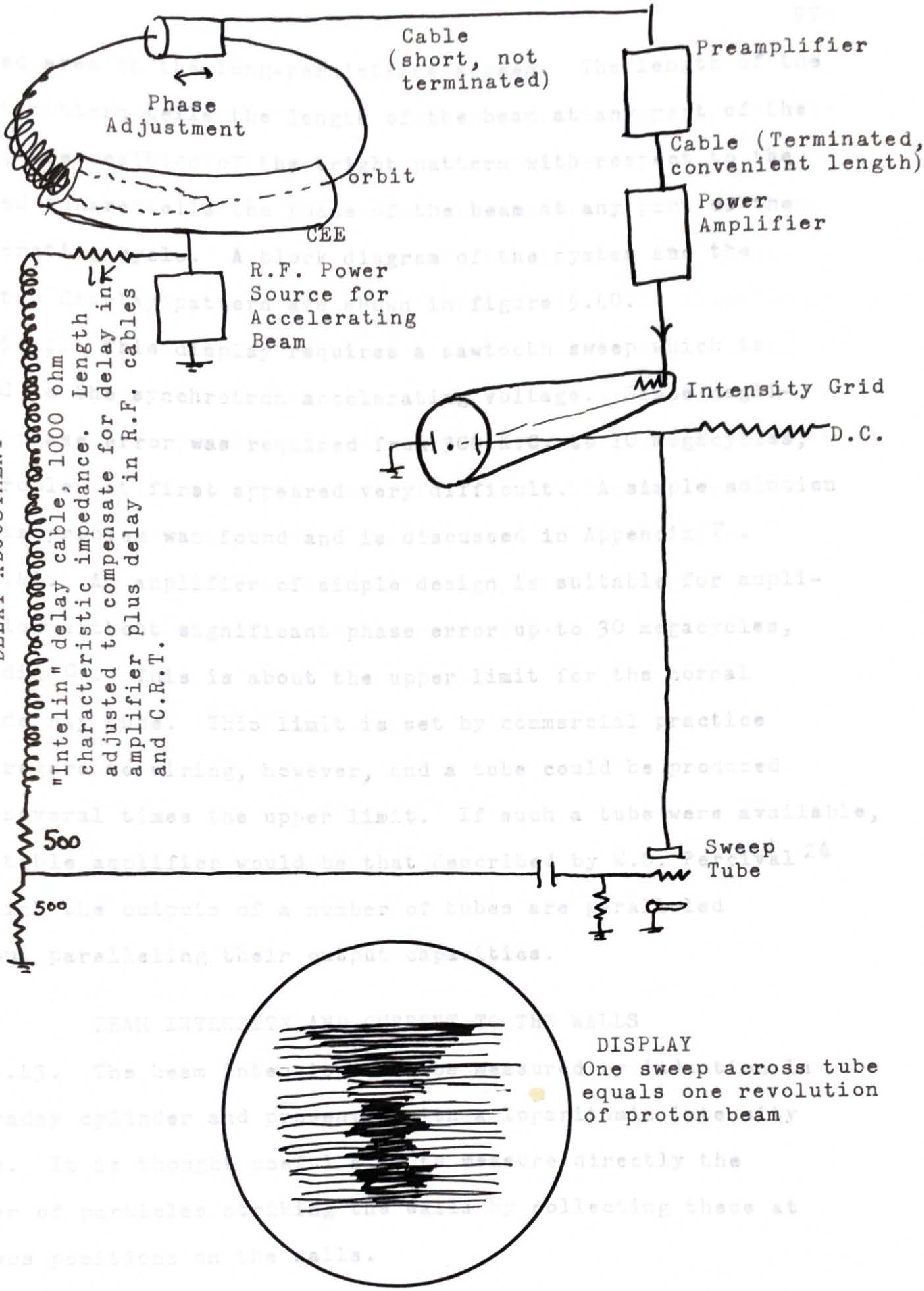

Cable
(short, not
terminated)
Phase
Adjustment
orbit
CEE
Preamplifier
Cable (Terminated,
convenient length)
Power
Amplifier
1K
R.F. Power
Source for
Accelerating
Beam
Intensity Grid
D.C.
DELAY ADJUSTMENT
"Intelin" delay cable, 1000 ohm
characteristic impedance. Length
adjusted to compensate for delay in
amplifier plus delay in R.F. cables
and C.R.T.
500
500
Sweep
Tube
DISPLAY
One sweep across tube
equals one revolution
of proton beam

FIGURE 5.40

excited area on the long-persistence screen. The length of the bright pattern tells the length of the beam at any part of the cycle, the position of the bright pattern with respect to the excited square tells the phase of the beam at any part of the accelerating cycle. A block diagram of the system and the expected display pattern are shown in figure 5.40.

5.41. This display requires a sawtooth sweep which is locked to the synchrotron accelerating voltage. Since negligible phase error was required from 300 K.C. to 10 megacycles, the problem at first appeared very difficult. A simple solution to this problem was found and is discussed in Appendix F.

5.42. An amplifier of simple design is suitable for amplification without significant phase error up to 30 megacycles, Appendix G. This is about the upper limit for the normal cathode ray tube. This limit is set by commercial practice with regard to wiring, however, and a tube could be produced with several times the upper limit. If such a tube were available, a suitable amplifier would be that described by W.S. Percival[24] in which the outputs of a number of tubes are paralleled without paralleling their output capacities.

BEAM INTENSITY AND CURRENT TO THE WALLS

5.43. The beam intensity will be measured by induction in a Faraday cylinder and presented with a logarithmic intensity scale. It is thought useful also to measure directly the number of particles striking the walls by collecting these at various positions on the walls.

SENSITIVITY OF THE DEVICES

5.44. For moderately long electrodes, the capacity and the charge induced are both proportional to the length of the electrode. Since the signal voltage is the ratio of these two, it is independent of length. It seems likely that the sensitivity of stray pickup from the R.F. oscillator will likewise be independent of length. For a band width of 10 megacycles, about 2×10^6 particles in the beam induce a voltage equal to tube noise. For position location to 1 per cent, 2×10^8 particles are required. The sensitivity of pickup plates on the walls depends upon their size and capacity to earth. The arrival of 500 particles as a pulse can just be detected under ideal conditions on a small electrode.

CHAPTER VI

MEASUREMENT OF BUNCH SHAPE IN THE CYCLOTRON

6.1. Interest in the measurement of bunch shape in the cyclotron dates from the suggestion of Professor Peierls that if the output of the cyclotron were found to consist of short pulses, shorter time scales would be available for use in the measurement of short-lived induced radioactivity and in delayed emission experiments. One of the interests of this laboratory has been the study of the high energy, fixed frequency cyclotron. Thus it was felt that the direct measurement of phases and bunch shapes throughout the accelerating process would add significantly to the information already acquired here on the operation of cyclotrons at high energy. A suitable method of carrying out this measurement has been evolved by the writer.

6.2. Although the electrical application of the stroboscopic principle which is used, has been in the public domain since 1870, none of the arrangements yet suggested could be applied to the present problem. The writer's apparatus can also be used conveniently for the measurement suggested by Professor Peierls. Such an application is being planned in this laboratory. Unfortunately, a limitation of the time available to him made it necessary for the writer to leave the measurements on the cyclotron to be completed by others in the laboratory. However, the measurements made by the

writer are sufficient to show that the cyclotron can produce, under conditions which have been defined, bunches of particles occupying about 20 electrical degrees or 5.5×10^{-9} seconds. These pulses are narrow enough to be useful in the measurement of short-lived radioactivity and in delayed emission experiments. It has been found that the phase of the particles is a nearly linear function of magnetic field. This provides a better relation for an automatic tuning scheme than any yet suggested.

6.3. In this chapter an approximate expression has been obtained for the phase of the particles during acceleration as a function of the orbit radius, input phase, magnetic field and dee voltage. Using this expression, a discussion is given of the dependence of the minimum voltage required for acceleration upon input and output phase and magnetic field. The range of output phases which can be covered by accelerated particles under various conditions has been found. The phase behaviour of the output has been shown to agree with the simple theory in certain respects and to disagree in others. A more complicated mechanism is suggested.

6.4. Although the fixed frequency cyclotron reached a state of great usefulness with a comparatively small amount of effort toward refinement, the attempt to extend it to proton energies beyond about 8 Mev. brings rapidly increasing difficulties. For some purposes, even above this limit, the fixed

frequency cyclotron because of its high output is still a valuable accelerator. Fremlin and Gooden[25] have shown that even for 300 Mev. deuterons the fixed frequency cyclotron provides the cheapest source per microampere. (They do not suggest the building of such a machine.) While early success in the low energy range was achieved with very little monitoring equipment, high energy machines require a very exact study of the mechanism of acceleration. In the low energy cyclotron, the magnetic field is essentially uniform and is set at the value which gives the greatest output. Most of the ions are accelerated near the crest of the R.F. wave. When, as in the higher energy machines, the mass change of the ion becomes significant, the phase of the ion must be made to shift to earlier phases at the beginning of acceleration in order to maintain the acceleration to as great a radius as possible, inspite of the retardation of phase caused by increasing mass. Where removal of the ions is desired, the limits of phase are still closer, since the ion must be receiving enough energy per revolution to separate the orbits. In order to use phases in which the ion crosses the gap before the peak of the wave, a magnetic focusing must be provided. This must offset the electric defocusing caused by the motion of the ion across the gap when the voltage is rising. To produce the necessary focusing, the field is required to change with radius in a direction which increases the effect due to increasing mass. A very careful planning of the magnetic field variation with radius is therefore essential. It is

fortunate that the nuclear resonance method, and others, make possible very accurate measurements of the field. The practical value of such control is shown by the success of the Birmingham cyclotron, both with regard to the high energy limit reached and the continuity of operation obtained at a slightly lower energy. It would evidently be of value in understanding the performance of a high energy machine if one could directly measure phases throughout the accelerating cycle, as well as the wave form of the output, that is, the range of phases which the particles have after acceleration.

BEHAVIOUR OF THE PARTICLE DURING ACCELERATION

6.5. Before describing the method evolved to make such measurements, we shall consider in greater detail the behaviour of the particle during acceleration. For an ion in a uniform magnetic field we have the equation

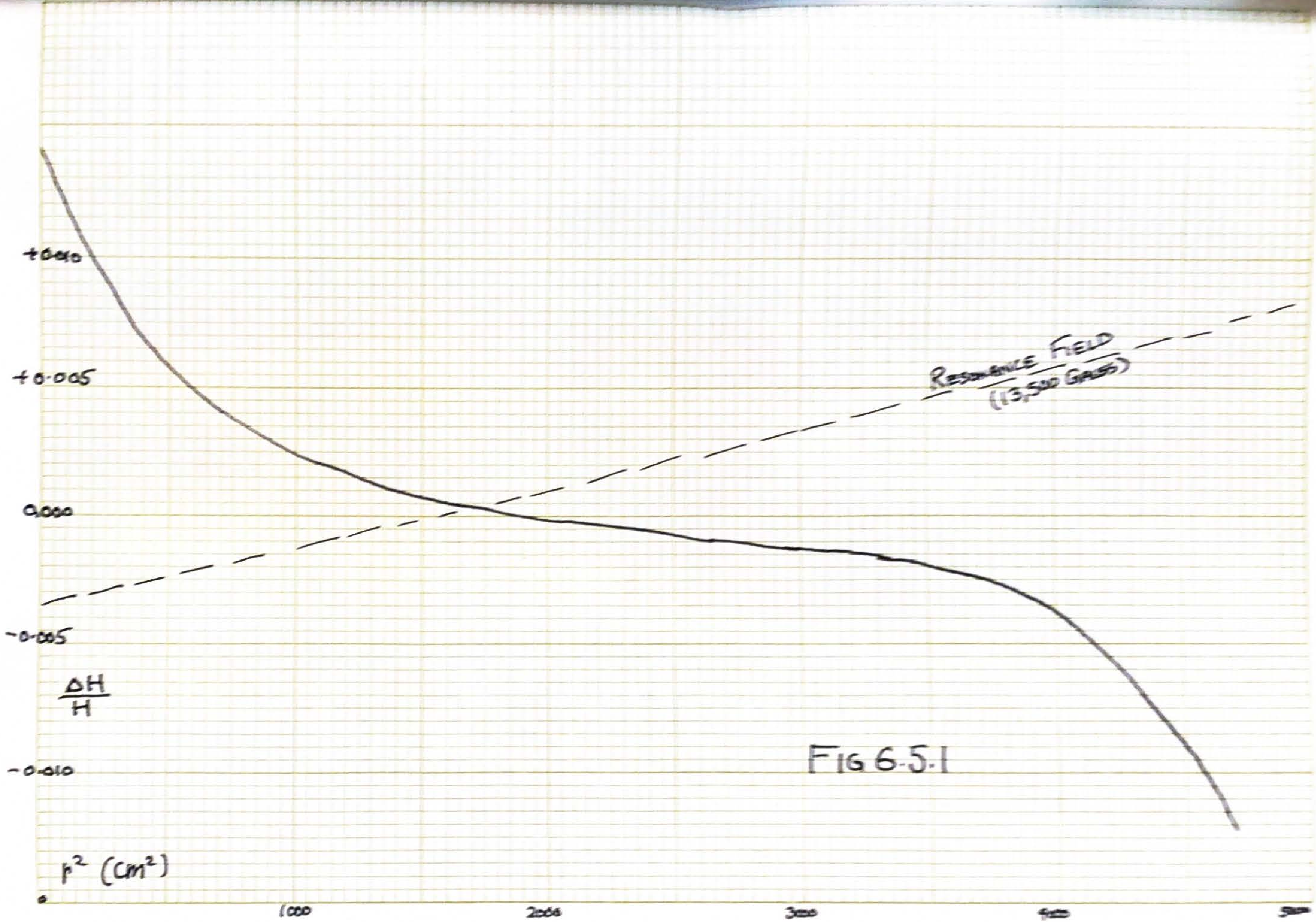
$$\frac{mv^2}{r} = \frac{Hev}{c} \tag{6.5.1}$$

m = mass of the ion
v = velocity of the ion
r = radius of the orbit
H = magnetic field
e = charge of the ion
c = velocity of light

or

$$H = \frac{v}{r}\,\frac{1}{ec}\,mc^2 \quad . \tag{6.5.2}$$

For 20 Mev. the mass of deuterons has increased by 1 per cent and the resonance field has increased by the same amount. When the resonance field is plotted against r^2, the result is a straight line, shown in figure 6.5. . In order to provide a



+0.010
+0.005
0.000
-0.005
-0.010
$\frac{\Delta H}{H}$
r^2 (cm²)
0
1000
2000
3000
4000
5000
RESONANCE FIELD
(13,500 GAUSS)

FIG 6.5.1

positive vertical focusing force at all radii, the magnetic field must decrease for increasing radius. The relative variation of magnetic field with radius in the Birmingham cyclotron[26] is shown in figure 6.5. . The angular velocity is proportional to the field (6.5.2). Thus the departure of the actual field from resonance value, expressed as a fraction of the resonance field, is equal to the departure of the angular velocity from its preferred value, expressed as a fraction of that value. That is, the difference between the two curves, (figure 6.5). is proportional to the rate of change of phase with angle. Expressed in symbols,

$$\frac{d\phi}{d\Theta} = f(r^2) \qquad (6.5.3)$$

ϕ = phase angle measured from the instant when the voltage is a maximum

Θ = total angle described by the particle

$f(r^2)$ = difference between the resonance field and the actual field as a fraction of the resonance field .

When the voltage gain per revolution is constant, the area between the two curves is proportional to the change in phase. In general, however, the energy gained, ΔE, per half revolution is not constant and

$$\Delta E = V \cos \phi \qquad (6.5.4)$$

V = peak R.F. voltage between the dees

E = particle energy in electron volts .

Representing the energy gain as taking place over the half revolution,

$$\frac{dE}{d\Theta} = \frac{V \cos \phi}{\pi} \quad \text{or} \quad E = \frac{1}{\pi}\int_0^{\Theta} V \cos \phi \, d\Theta \, . \qquad (6.5.5)$$

Since $\frac{1}{2}mv^2 = eE$, it follows from (6.5.2) that

$$r^2 = KE \quad \text{where} \quad K = \frac{mc^2}{eH^2\, 150} \quad . \tag{6.5.6}$$

For deuterons

$$K = 2.30 \times 10^{-4} \; . \tag{6.5.7}$$

We can now use (6.5.5) and (6.5.3) to obtain ϕ as a function of E and hence of r^2:

$$\frac{d\phi}{dE} = \frac{d\phi}{d\theta} \cdot \frac{d\theta}{dE}$$

$$= f(r^2)\,\frac{\pi}{V \cos \phi} \; .$$

Hence,

$$V \sin \phi - V \sin \phi_o = \pi \int_{E_o}^{E} f(KE)\, dE$$

or

$$\sin \phi = \frac{\pi}{KV} \int_{r_o^2}^{r^2} f(r^2)\, d(r^2) \; + \sin \phi_o \tag{6.5.8}$$

6.6. It may be seen that the phase of the particles arriving at a particular value of radius where

$$\int_{r_o^2}^{r^2} f(r^2)\, d(r^2) = 0$$

is the same as the entrance phase ϕ_o, and is independent of accelerating voltage, and thus independent of the number of revolutions performed in arriving at this radius. Also, for a particular value of r where

$$\int_{r_o^2}^{r^2} f(r^2)\, d(r^2) \neq 0$$

varying the dee voltage causes a change in phase. Increasing the dee voltage shifts the phase toward the entrance phase, and decreasing the dee voltage shifts the phase away from the

entrance phase. When the dee voltage is sufficiently low, the value of $\sin\phi$ may exceed one over a range of values of r^2. In this region there is no real value of ϕ which satisfies equation (6.5.8). This means that a particle starting with the given entrance phase cannot be accelerated in this region. When ϕ becomes real at still greater radii, it means that particles would be accelerated if placed at the inner limit of the real region with the appropriate phase and energy. The condition for particles to be accelerated out to the extraction radius is, of course, that ϕ must have real values for all values of r^2 from zero to the extraction radius.

6.7. The situation can be pictured graphically by a plot of $\sin\phi$ as a function of r^2. On this plot the variation of entrance phase corresponds to translating the curves parallel to the $\sin\phi$ axis. The variation of dee voltage corresponds to a change of scale on the $\sin\phi$ axis. Increasing the dee voltage expands the scale of the $\sin\phi$ axis, that is, the deviations in phase during acceleration are smaller with a high dee voltage than with a low dee voltage. We may draw two bounds:

$$\sin\phi = 1, \quad \sin\phi = -1 .$$

ϕ is considered positive when the particle crosses the gap before the voltage of the accelerating particle is a maximum. That is, ϕ, and hence $\sin\phi$, is positive for leading phases. As stated in section 6.6, conditions which are represented by $\sin\phi$ versus r^2 curves which cross either bound do not result in the acceleration of particles to the extraction radius.


FIG 6.7.1.
V_1
V_2
V_3
V_4
1.0
+0.8
+0.4
0
-0.4
-0.8
-1.0
1000
2000
3000
4000
r^2 (cm)^2
sinϕ

CRITICAL ACCELERATING VOLTAGE

6.8. The resonance value of magnetic field is the value at which acceleration can be obtained for the smallest value of dee voltage. Changing the magnetic field changes the shape of the integral in equation (6.5.8), namely

$$F(r^2) = \int_{r_o^2}^{r^2} f(r^2)\,d(r^2) , \qquad (6.8.1)$$

and therefore the shape of the sin ϕ curve. It should be noted that since the rate of change of phase is always decreasing algebraically, the sin ϕ curve is concave downward in the interval. Thus the function $F(r^2)$ is initially increasing, passes through a single maximum, then decreases in the remainder of the interval (figure 6.8.1). To find the critical accelerating voltage it is thus necessary to find the value of entrance phase ϕ_o and magnetic field h which give the minimum value of V under the condition that ϕ is everywhere real. The condition may be more readily set up by referring to the foregoing plot (figure 6.7.1). Let $F_c(r^2)$ be the value of $F(r^2)$ which will result in the lowest V. It is clear that $F_c(r^2)$ is that function F which has the smallest distance d between its maximum and minimum values in the accelerating range. We will now show that $F_c(r^2)$ is zero at the extraction radius. The meaning of this condition can be seen from the following sketch, figure 6.8.1. The line AB divides $F(r^2)$ into two parts. On the left of AB, $F(r^2)$ is increasing, that is the

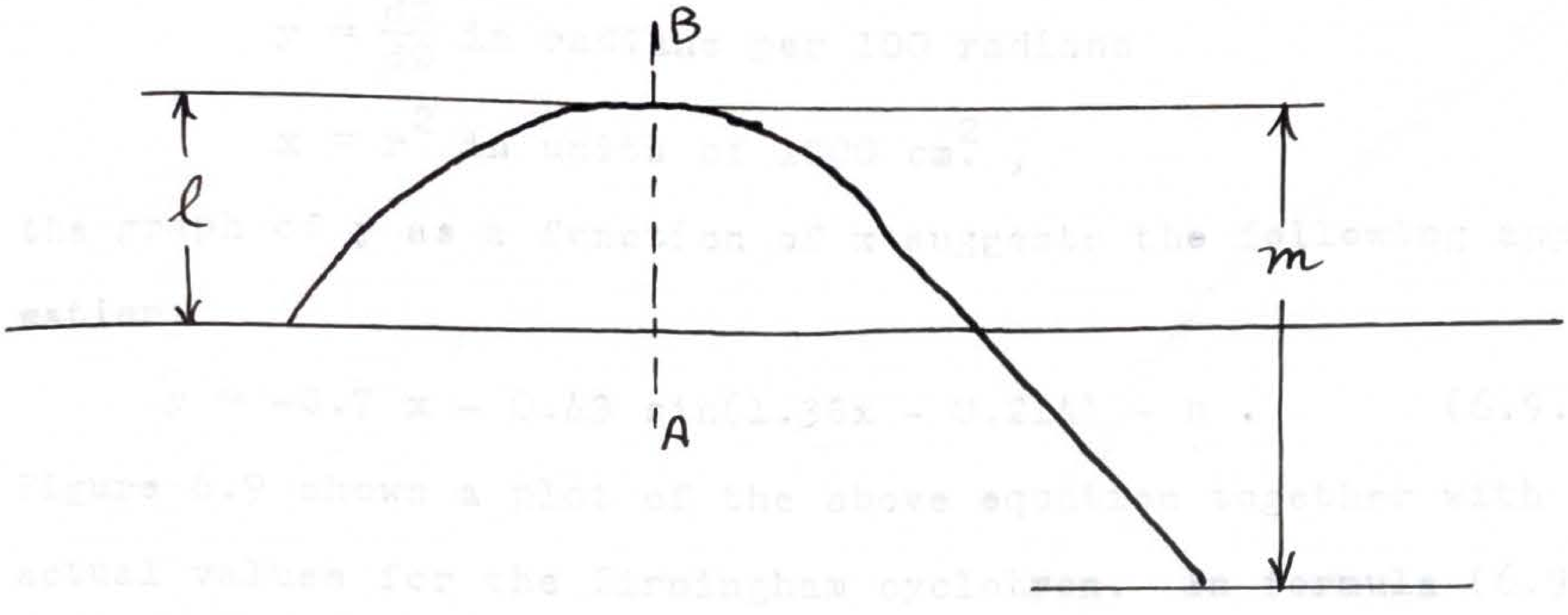


FIGURE 6.8.1

phase is changing toward leading values. On the right of AB, $F(r^2)$ is decreasing, that is, the phase is changing toward lagging values. When the magnetic field is changed, one part of the integral is increased and the other is decreased. Suppose $m > \ell$. That is, m is the distance d between the maximum and minimum values of $F(r^2)$ in the interval. An adjustment of h which decreases m and increases ℓ will now decrease d. This continues to hold until $\ell = m$. Thus the smallest distance between the maximum and minimum values of $F(r^2)$ in the interval occurs when $\ell = m$, that is when $F(r^2) = 0$ at the end of the interval. This means that

$$d = \left[F_c(r^2) \right]_{max} . \qquad (6.8.2)$$

APPROXIMATION FOR THE FUNCTION $f(r^2)$

6.9. To study in greater detail the effect of the variables V, ϕ_o, and h on the acceleration process, it is desirable to find

an approximate expression for $f(r^2)$. Where

$$y = \frac{d\phi}{d\theta} \text{ in radians per 100 radians}$$

$$x = r^2 \text{ in units of 1000 cm}^2 ,$$

the graph of y as a function of x suggests the following approximation:

$$y = -0.7\ x - 0.43 \sin(1.38x - 0.214) - h \ . \qquad (6.9.1)$$

Figure 6.9 shows a plot of the above equation together with the actual values for the Birmingham cyclotron. In formula (6.9.1) h is the field expressed in per cent above an arbitrary value (close to resonance) 13,500. In the work that follows, it will be assumed that $r_o = 0$, that is, the particles start at the centre. We have the relation:

$$F(r^2) = \int_0^{r^2} f(r^2)\,d(r^2) = 10 \int_0^{x} y\ dx = 10\ G(x) . \qquad (6.9.2)$$

Using (6.9.1), we obtain for G(x)

$$G(x) = \int_0^x y\ dx$$

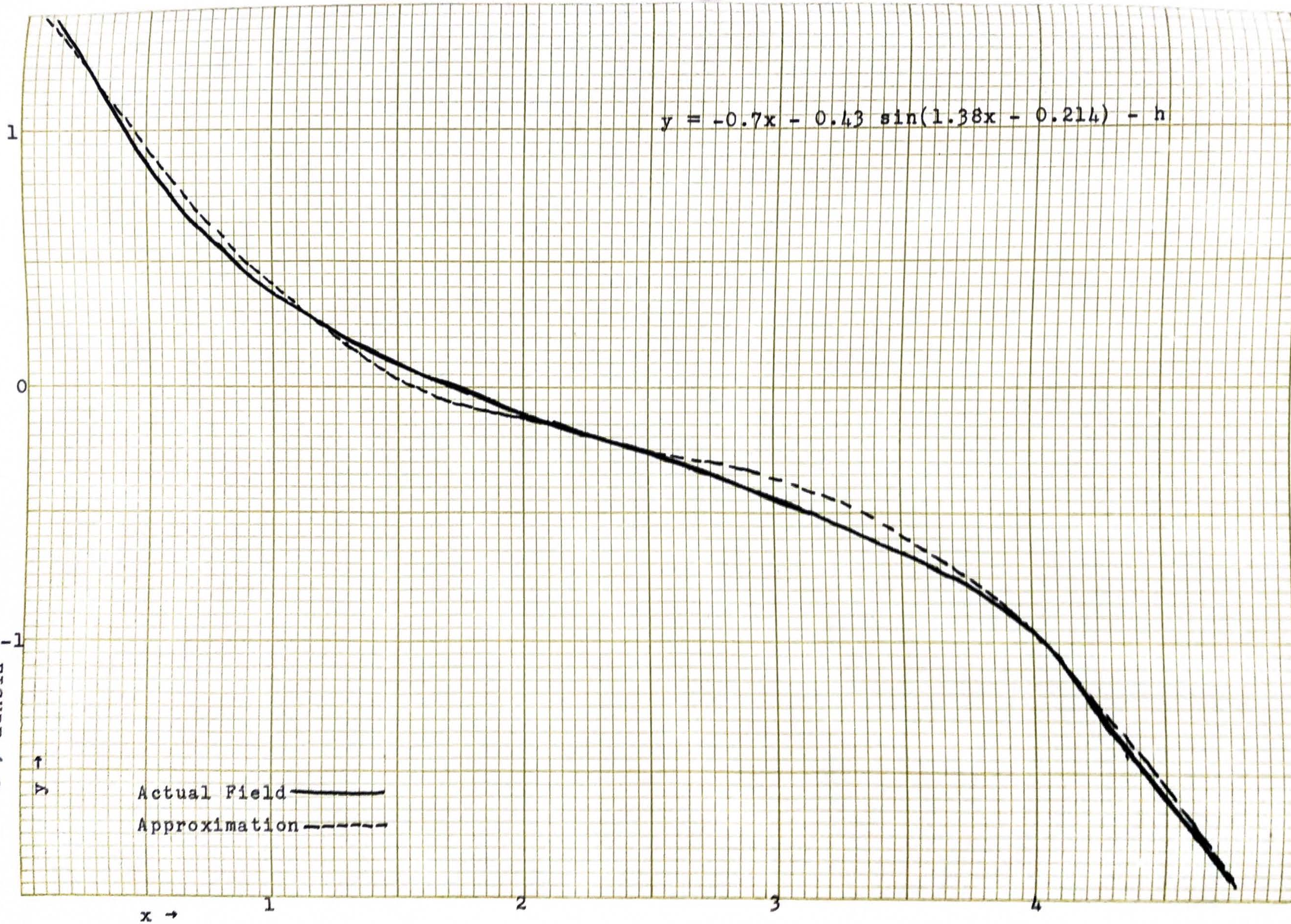
$$= -0.35\ x^2 - hx + 0.311 \cos(1.38x - 0.214) - 0.304 \qquad (6.9.3)$$

and

$$\sin \phi = \frac{10\ \pi}{KV}\Big[- 0.35x^2 - hx + 0.311\cos(1.38x - 0.214) - 0.304\Big] + \sin \phi_o \ . \qquad (6.9.4)$$

The value of K has been given, (6.5.1),

$$K = 2.3 \times 10^{-4} \ .$$



$y = -0.7x - 0.43 \sin(1.38x - 0.214) - h$
1
0
-1
y →
Actual Field
Approximation
x →
1
2
3
4

FIGURE 6.9

MINIMUM VOLTAGE REQUIRED TO ACCELERATE PARTICLES

TO THE EXTRACTION RADIUS

6.10. We can now find the value of h which makes $F(r^2)$ equal to zero for $r^2 = 4700$, that is, $G(x) = 0$ for $x = 4.7$. We obtain

$$h = -1.675 \; . \tag{6.10.1}$$

This is the resonance value of h. The expression for $G(x)$ at this value of h is

$$G_r(x) = -0.35x^2 + 1.675x + 0.311\cos(1.38x-0.214) - 0.304 \; . \tag{6.10.2}$$

The value of x at which $G_r(x)$ is a maximum is

$$x \cong 2.25 \; . \tag{6.10.3}$$

The maximum value of $G_r(x)$ is

$$G_r(2.25) = 1.389 \; , \tag{6.10.4}$$

that is, the maximum value of $F_r(r^2)$ for the resonance value of h is

$$d = F_r(2250) = 13.89. \tag{6.10.5}$$

We now choose the entrance phase so that the phase curve for the resonance value of h reaches both limits $\sin\phi = 1$, $\sin\phi = -1$ for a particular value of voltage. That is

$$\sin\phi_o = -\frac{\pi}{KV}\, 6.945$$

and the phase curve is

$$\sin\phi = \frac{\pi}{KV}\left[-0.35x^2 + 1.675x + 0.311\cos(1.38x-0.214) - 7.249\right].$$

Now the particular value of voltage for which $\sin\phi = 1$ at its maximum value is found from

$$1 = \frac{\pi}{KV}\, 6.945$$

that is,

$$V = 94{,}860 \quad . \tag{6.10.6}$$

The minimum value of V is thus 94,860, and the smallest value of R.F. voltage between each dee and ground which can accelerate particles to the extraction radius (critical voltage) is approximately 47 K.V.

RESTRICTIONS ON OUTPUT AND INPUT PHASE

6.11. Since the critical situation arrived at in the preceding section gives orbits with zero spacing at the extraction radius, it is necessary to set a smaller negative limit on $\sin\phi$. A suitable value might be $\sin\phi = -.866$ at which the spacing of the orbits is half that for $\sin\phi = 0$. Introducing this limit without changing $F_r(r^2)$, makes the minimum voltage required to extract particles just under 51 K.V. It is clear, on the consideration of phase limitations during acceleration that particles which are to be accelerated at the lowest voltages must start with an extreme lagging phase. This is evident from the fact that a particle which starts with an extreme lagging phase will reach the upper bound at a greater radius than that for a particle with any other phase. Thus the phase change in the latter part of the acceleration will be smaller than that of a particle which starts with any earlier phase. Assuming a uniform field, Allwood[27] and Bohm and Foldy[28] have suggested from theoretical considerations that particles starting with all possible phases tend to become, within the first revolution, bunched about a single phase. Since no experiments have yet been done to confirm these

assertions, and since the exact conditions which obtain in the vicinity of the ion source are not precisely defined, we should investigate the results of a number of possible assumptions regarding the limitations placed on entrance phase by the nature of the ion source, with a view to determining which of these agrees best with the observed characteristics of the acceleration process.

DEPENDENCE OF CRITICAL VOLTAGE ON INPUT PHASE LIMITATIONS

6.12. We can set up a mechanism for obtaining the critical voltage V as a function of the limiting initial phase, ϕ_o. Let ϕ_L designate the extreme lagging output phase and ϕ_o the extreme lagging input phase. Since the lowest voltage limit is achieved by starting at the most extreme lagging input phase and leaving at the most extreme lagging output phase, we obtain

$$\frac{10\pi}{KV} G(h, 4.7) + \sin \phi_o = \sin \phi_L \; . \qquad (6.12.1)$$

The phase curve, sin ϕ as a function of r^2, is uniquely determined when we add the condition that its maximum be at the upper bound, sin ϕ = 1. That is,

$$1 = \frac{10\pi}{KV} G(h, x_{max}) + \sin \phi_o \; . \qquad (6.12.2)$$

Here we have extended the notation of section 6.9 to show the dependence of G on both h and x by writing

$$G(x) \equiv G(h,x) \; . \qquad (6.12.3)$$

The notation x_{max} denotes the value of x at which G(h,x) is a maximum. This depends upon the value of h. Thus x_{max} is a function of h, determined implicitly by the equation

$$-0.7x_{max} - 0.43\sin(1.38x_{max} - 0.214) - h = 0. \qquad (6.12.4)$$

TABLE 6.12.1

$$G(h,x) = -0.35x^2 - hx + 0.311\cos(1.38x-0.214) - 0.304$$

x	G(h,x) +hx	G(-1.45,x)	G(-1.5,x)	G(-1.55,x)	G(-1.6,x)	G(-1.65,x)	G(-1.7,x)	G(-1.75,x)	G(-1.8,x)	G(-1.85,x)	G(-1.9,x)	G(-1.95,x)	G(-2,x)
0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.25	-0.018	0.345	0.358	0.370	0.382	0.395	0.408	0.420	0.432	0.445	0.458	0.470	0.482
0.50	-0.116	0.610	0.635	0.660	0.685	0.710	0.735	0.760	0.785	0.810	0.835	0.860	0.885
0.75	-0.289	0.799	0.835	0.873	0.911	0.948	0.986	1.023	1.061	1.098	1.136	1.173	1.211
1.00	-0.532	0.918	0.968	1.018	1.068	1.118	1.168	1.218	1.268	1.318	1.368	1.418	1.468
1.25	-0.833	0.980	1.043	1.105	1.168	1.230	1.293	1.355	1.418	1.480	1.543	1.605	1.668
1.50	-1.179	0.996	1.071	1.146	1.221	1.296	1.371	1.446	1.521	1.596	1.671	1.746	1.821
1.75	-1.559	0.979	1.067	1.154	1.242	1.329	1.417	1.504	1.592	1.679	1.767	1.854	1.942
2.00	-1.962	0.938	1.038	1.138	1.238	1.338	1.438	1.538	1.638	1.738	1.838	1.938	2.038
2.25	-2.379	0.884	0.997	1.109	1.222	1.334	1.446	1.559	1.671	1.784	1.897	2.009	2.121
2.50	-2.800	0.825	0.950	1.075	1.200	1.325	1.450	1.575	1.700	1.825	1.950	2.075	2.200
2.75	-3.231	0.756	0.894	1.031	1.169	1.306	1.444	1.581	1.719	1.856	1.994	2.131	2.269
3.00	-3.673	0.677	0.827	0.977	1.127	1.277	1.427	1.577	1.727	1.877	2.027	2.177	2.327
3.25	-4.137	0.576	0.739	0.901	1.064	1.225	1.388	1.550	1.713	1.875	2.038	2.200	2.363
3.50	-4.622	0.453	0.628	0.803	0.978	1.153	1.328	1.503	1.678	1.853	2.028	2.203	2.378
3.75	-5.151	0.286	0.474	0.661	0.849	1.036	1.224	1.411	1.599	1.786	1.974	2.161	2.349
4.00	-5.726	0.074	0.274	0.474	0.674	0.874	1.074	1.274	1.474	1.674	1.874	2.074	2.274
4.25	-6.374	-0.212	0.001	0.213	0.426	0.638	0.851	1.063	1.276	1.488	1.701	1.913	2.125
4.50	-7.110	-0.585	-0.360	-0.135	0.090	0.315	0.540	0.765	0.990	1.215	1.440	1.665	1.890
4.70	-7.875	-1.060	-0.825	-0.590	-0.355	-0.120	0.115	0.350	0.585	0.820	1.055	1.290	1.525

For any given h, the corresponding value of x_{max}, and of $G(h,x_{max})$ can be found conveniently from Table 6.12.1. Since x_{max} is a function of h, it is not possible to eliminate h explicitly from equations (6.12.1) and (6.12.2). The value of V corresponding to a given value of ϕ_o can, however, be found simply with the aid of Table 6.12.1. Table 6.12.2 shows the value of critical voltage for various limiting input phases.

CRITICAL VOLTAGE
AS A FUNCTION OF LIMITING INPUT PHASE

ϕ_o	V	V/2
-0°	144	72
-10°	130	65
-20°	122	61.5
-30°	115	57.5
-40°	110	55
-50°	105	52.5
-60°	102	51
-90°	99	49.5

TABLE 6.12.2

DEPENDENCE OF h_r ON INPUT PHASE LIMITATIONS

6.13. Equations (6.12.1) and (6.12.2) give the value of magnetic field at which acceleration can be obtained for the smallest value of dee voltage. If we now solve these equations for magnetic field, eliminating V instead of h, we obtain a relationship between h_r and ϕ_o:

$$\frac{G(h_r,x_{max})}{G(h_r,4.7)} = - \frac{1 - \sin \phi_o}{-\sin \phi_L + \sin \phi_o} \qquad (6.13.1)$$

In Table 6.13, ϕ_L is assumed to be $-60°$.

h_r AS A FUNCTION OF ϕ_o

ϕ_o	h_r
$-0°$	-1.48
$-10°$	-1.53
$-20°$	-1.575
$-30°$	-1.61
$-40°$	-1.64
$-50°$	-1.66
$-60°$	-1.675
$-90°$	-1.7

TABLE 6.13

DEPENDENCE OF CRITICAL VOLTAGE UPON h (RESONANCE CURVES)

6.14. The critical voltage when h is disposable has been shown to be associated with the phase curve passing through the extreme lagging input phase limit and the extreme lagging output phase limit. The particular value h_r of h which produces this condition has been called the resonance value of h. From the definition of h_r it is clear that varying h away from h_r in either direction will increase the critical voltage. When we wish to examine the acceleration process for magnetic fields different from the resonance field, we must redefine the conditions for obtaining the critical voltage. The way in which this is done will depend upon the limits imposed on

input phases. In order to cover all cases, we should consider an upper limit as well as a lower limit on ϕ_o:

$$\phi_{o\ell} \leq \phi_o \leq \phi_{ou} \tag{6.14.1}$$

In addition to touching the upper bound, $\sin \phi = 1$, the phase curve representing the critical voltage will always pass through one of the input phase limits or the output phase limit. This may be shown as follows. Consider some phase curve which lies within the upper bound but does not pass through any of the limits. A decrease in the value of V may be made together with an appropriate adjustment of ϕ_o, therefore the curve was not the critical curve. Although the critical voltage is a continuous function of h, it is necessary to distinguish between two cases: (1) $h < h_r$, (2) $h > h_r$. When h has a larger negative value, either the resonance field has decreased or the actual field has increased. For case (1), $h < h_r$, the critical phase curve for any h passes through the lower limit on input phase, and V is determined by the condition that this curve shall not cross the upper bound $\sin\phi = 1$:

$$\frac{10\,\pi}{KV}\, G(h,x_{max}) = 1 - \sin \phi_{o\ell}\ ,$$

$$V = \frac{10\pi}{K(1 - \sin \phi_{o\ell})}\, G(h,x_{max})\ . \tag{6.14.2}$$

Case (2), $h > h_r$: For fields very near the resonance value, the critical phase curve is determined by the condition that it pass through the limit on output phase:

$$\frac{10\,\pi}{KV}\, G(h,x_{max}) - \frac{10\,\pi}{KV}\, G(h,4.7) = 1 - \sin \phi_L \tag{6.14.3}$$

As h increases, the initial point of the critical phase curve reaches the upper limit of input phase. Since as h is increased the curve moves away from the upper bound of sin ϕ, the limiting curve is determined by the condition that it shall pass through the upper input phase limit and the output phase limit. That is,

$$\frac{10\,\pi}{KV}\,G(h,4.7) = \sin\phi_L - \sin\phi_{ou} \qquad (6.14.4)$$

6.15. Figure 6.15 is a plot of the critical voltage versus h, with ϕ_{ou} and $\phi_{o\ell}$ as parameters, and $\phi_L = -60^\circ$. It should be noted that equation (6.14.3) gives a single curve independent of ϕ_{ou} or $\phi_{o\ell}$. The resonance curve corresponding to a given restriction on input phase:

$$\phi_{o\ell} \leqq \phi_o \leqq \phi_{ou}$$

is found by starting at the left and proceeding down the curve corresponding to $\phi_{o\ell}$ until it intersects the curve of equation (6.14.3). The curves intersect at the resonance value of h. From the resonance point one proceeds up the curve of equation (6;14.3) until its intersection with the curve corresponding to ϕ_{ou}. This is the complete resonance curve for the condition stated. Two typical resonance curves A and B have been indicated. A is for $\phi_{o\ell} = -20^\circ$, $\phi_{ou} = 0^\circ$ and B is for $\phi_{o\ell} = -50^\circ$, $\phi_{ou} = -24^\circ$.

V, KILOVOLTS
h
-2·3
-2·1
-2·0
-1·9
-1·8
-1·7
-1·6
-1·5
-1·4
-1·3
100
110
120
130
140
150
ϕ_L
ϕ_U
-10°
-20°
-30°
-40°
-50°
-60°
-90°

FIG 6·15

VARIATION OF THE OUTPUT PHASE RANGE WITH VOLTAGE AND MAGNETIC FIELD

6.16. Case I. Let us assume that the magnetic field is given the resonance value (see section 6.8) and that there are no restrictions on input and output phases. That is, we take $h = -1.675$ (formula (6.10.1)) and $G(-1.675, 4.7) = 0$. Thus equation (6.9.4) at $x = 4.7$ becomes

$$\sin \phi\Big]_{x=4.7} = \sin \phi_o \tag{6.16.1}$$

There is no output at all for a voltage below the critical value. At the critical voltage, only particles entering with a phase of -90° can be accelerated to the extraction radius. As the voltage is raised above the critical value, the range of input phases over which particles can be accelerated to the extraction radius increases. The critical phase, that is, the most positive phase for which acceleration is possible, is determined as a function of voltage by the requirement that $\sin \phi$ must be less than 1 over the whole range of radii. That is:

$$1 \geq \frac{10\,\pi}{KV} G(-1.675, x_{max}) + \sin \phi_o \tag{6.16.2}$$

CASE I - TABLE 6.16

V/2	ϕ range (degrees)			Width (Degrees)
45				0
50	-64.2	to	-90	25.8
55	-46.7		-90	43.3
60	-35.6		-90	54.4
65	-27.5		-90	62.5
70	-21.0		-90	69.0
75	-15.5		-90	74.5
80	-10.8		-90	79.2
85	- 6.8		-90	83.2
90	- 3.3		-90	86.7
95	- 0.0		-90	90.0

Table 6.16 shows the variation of output phase range with voltage. It can be seen that the assumptions of Case I result in an output phase range which is zero at the critical voltage and increases, rapidly at first then more slowly, with increasing V. At 80 K.V. from dee to ground, the highest voltage which can be reached conveniently in the Birmingham cyclotron, the output phase range, on the assumptions of Case I, would be 79°. Another characteristic feature of the variation of the output phase range with voltage in Case I is that the negative phase limit is constant and as the voltage is raised the phase ranges are extended from this limit in the positive direction.

6.17. Case II. If we introduce a restriction on the output phase as was done in section 6.11, that is, that the sine of the output phase cannot have a negative value numerically larger than .866, we get the relation between output phase range and voltage shown in Table 6.17.

CASE II

V/2	ϕ range (degrees)			Width (Degrees)
50				0
55	-46.7	to	-60	13.3
60	-35.6		-60	24.4
65	-27.5		-60	32.5
70	-21.0		-60	39.0
75	-15.5		-60	44.5
80	-10.8		-60	49.2
85	- 6.8		-60	53.2
90	- 3.3		-60	56.7
95	0		-60	60.0

TABLE 6.17

The assumptions in case II result in an output phase range the variation of which with voltage is of the same general nature as that obtained in case I. The critical voltage, however, is slightly higher and the rate of increase of the phase range at voltages near the critical value is not so great. At 80 K.V. the width is 49°.

6.18. Case III. With the assumptions of case II regarding limits on input and output phase, we now examine two values of magnetic field, one on each side of the resonance value. The equations describing the situation are

$$1 - \frac{10\,\pi}{KV}\, G(h, x_{max}) \geqq \sin \phi_o \qquad (6.18.1)$$

and

$$\sin \phi \Big]_{x=4.7} = \frac{10\,\pi}{KV}\, G(h, 4.7) + \sin \phi_o \,. \qquad (6.18.2)$$

Equation (6.18.1) imposes the condition that only those values of ϕ_o can be considered for which the curves remain below the limit $\sin \phi = 1$. When the field is below the resonance value, the variation of the output phase range is again similar to that obtained in Case I and Case II, the critical voltage being higher than that for either Case I or Case II. The output phase ranges obtained for $h = -1.55$ are shown in Table 6.18.1. When the field is above the resonance value, the range of output phases obtained immediately above the critical voltage increases with critical voltage as before. The ranges lie within the permissible range of output phases until the increase in width is sufficient to bring the edge of the range to -60°. Thereafter, the range increases with this as a fixed limit in the negative direction. Table 6.18.2 shows the situation for $h = -1.8$.

CASE III, h = -1.55

V/2	ϕ range (degrees)			Width (degrees)
64	-59.4	to	-60	0.6
65	-56.4		-60	3.6
70	-44.6		-60	15.4
75	-36.0		-60	24.0
80	-29.3		-60	30.7
85	-23.7		-60	36.3
90	-18.9		-60	41.1
95	-14.7		-60	45.3
100	-11.0		-60	49.0

TABLE 6.18.1

CASE III, h = -1.8

V/2	ϕ range (degrees)			Width (degrees)
59	-18.8	to	-18.8	0
60	-17.4		-19.5	2.1
65	-11.5		-22.7	11.2
70	- 6.6		-25.4	18.8
75	- 2.3		-27.9	25.6
80	1.5		-30.4	31.9
85	4.7		-32.0	36.7
90	7.7		-33.8	41.5
95	10.3		-35.4	45.7
100	12.7		-36.9	49.6
300	47.7		-60	107.7

TABLE 6.18.2

6.19. Case IV. The variation of the output phase range with magnetic field will now be considered, again retaining the assumptions of case II regarding input and output phases. We have the equations (6.18.1) and (6.18.2) of Case III, in which V is now given a definite value and the phase range is considered as a function of h. For V = 150, the equations are

$$1 - 0.91\ G(h,x_{max}) \geqq \sin \phi_o$$

$$\sin \phi \Big]_{x=4.7} = 0.91\ G(h,4.7) + \sin \phi_o \ .$$

Table 6.19 shows the variation of output range with h for V = 150.

CASE IV

h	φ range (degrees)			Width (Degrees)
-1.95	6.1			0
-1.90	6.1	to	- 2.3	8.4
-1.85	2.1		-14.7	16.8
-1.80	-2.3		-27.9	25.6
-1.75	-7.0		-43.0	36.0
-1.70	-12.4		-60	47.6
-1.65	-19.1		-60	40.9
-1.60	-26.9		-60	33.1
-1.55	-35.9		-60	24.1
-1.50	-47.3		-60	12.7
-1.45	-58.9		-60	1.1
-1.44	-60		-60	0

TABLE 6.19

When h is varied with V constant and input phase range unrestricted, the length of the bunch is greatest near the resonance value of field. The length shrinks on either side of resonance. On the

low field side, the phase ranges have a common limit in the negative direction. The centre of the bunch shifts in the negative direction as the field goes from high to low values. The maximum phase range is about 48° for V = 150 K.V.

6.20. <u>Case V</u>. When a supply of ions with a negative phase limit is considered, a lower bound different from -1 is imposed on sin ϕ_{o}. The conclusions reached in Cases II, III, IV are not altered qualitatively. The critical voltage is raised and the width of the pulses for a given voltage is decreased. This effect is illustrated in Table 6.20. A fixed lower bound of -0.5 is placed on sin ϕ_{o}, and the magnetic field is varied as in Case IV, section 6.19.

CASE V

h	ϕ range (degrees)			Width (Degrees)
-1.80				0
-1.75	- 7.0	to	-10.5	3.5
-1.70	-12.4		-23.3	10.9
-1.65	-19.1		-37.5	18.4
-1.60	-26.9		-55.4	28.5
-1.55	-35.9		-60	24.1
-1.50	-47.3		-60	12.7
-1.45	-58.9		-60	1.1
-1.44	-60		-60	0

TABLE 6.20

6.21. Case VI. The variation of the output phase range with magnetic field will now be considered on the assumption that the input phase has an upper as well as a lower limit. The equation

$$\sin \phi = \frac{10\,\pi}{KV}\,G(h,x) + \sin \phi_o \qquad (6.21.1)$$

again determines the phase of a particle starting with input phase ϕ_o. Equation

$$1 - \frac{10\,\pi}{KV}\,G(h,x_{max}) \geq \sin \phi_o \qquad (6.21.2)$$

determines whether the particle with initial phase ϕ_o can be accelerated to the extraction radius. If $\Delta\sin\phi_o$ represents a range of permissible input phases, we have

$$\Delta\sin\phi = \Delta\sin\phi_o \; . \qquad (6.21.3)$$

If (6.21.2) is satisfied, $\Delta\sin\phi\big]_{x=4.7}$ characterises the range of output phases corresponding to $\Delta\sin\phi_o$. Equation (6.21.3) is true throughout the cyclotron, that is for all values of x, and is independent of the actual values of the input phases. Evidently, the degree range corresponding to a given $\Delta\sin\phi_o$ (or $\Delta\sin\phi$) depends upon the actual value of ϕ_o (or ϕ). This effect constitutes a bunching when ϕ approaches zero and a debunching when ϕ approaches 90°. It may also be pointed out that for a fixed $\Delta\sin\phi$, the output range expressed in degrees becomes greater as the pulse moves toward later phases.

6.22. The variation of magnetic field affects the actual value of ϕ at extraction corresponding to a given ϕ_o according to equation (6.21.1) with $x = 4.7$. This has the effect on the width of the output phase range discussed above. Table 6.22 illustrates this situation where $V = 150$, $h = -1.63$ and $\Delta\sin\phi_o = 0.36$.

CASE VI

h	Outside Edge of Output Pulse	Peak of Output Pulse
-1.63	-74°	-53°
-1.66	-56.4	-42.1
-1.69	-44.8	-32.9
-1.72	-35.3	-24.5
-1.75	-26.7	-16.7
-1.78	-18.7	- 9.2

TABLE 6.22

To facilitate comparison with the experimental work, the values of h have been spaced more closely in this table than in the preceding ones.

6.23. CASE VII. The same considerations apply in the discussion of the variation of output phase range with voltage under the assumption that

$$\phi_{o\ell} \leqq \phi_o \leqq \phi_{ou} \ .$$

We have again the equations

$$\sin \phi = \frac{10\ \pi}{KV} G(h,x) + \sin \phi_o \tag{6.23.1}$$

and

$$1 - \frac{10\ \pi}{KV} G(h,x_{max}) \geqq \sin \phi_o \ . \tag{6.23.2}$$

It should be noted in addition that, if G(h,x) is zero, there is no shift of output phase with voltage and the shift with voltage becomes progressively more marked as G(h,x)

moves away from zero. Table 6.23 shows the variation of output phase range with input voltage for h = -1.66, $\phi_{o\ell} = -50^{\circ}$ and $\phi_{ou} = -24^{\circ}$.

CASE VII

V/2	ϕ range (degrees)			Width (degrees)
52.5	-60	to	-57.7	2.3
55	-59		-49.8	9.2
60	-58.1		-38.0	20.1
65	-57.5		-29.5	28.0
70	-56.8		-28.6	28.3
75	-56.3		-28.1	28.2
80	-55.9		-28.0	27.9
85	-55.6		-27.7	27.9
90	-55.2		-27.5	27.7
95	-54.9		-27.3	27.6

TABLE 6.23

METHOD OF MEASURING CYCLOTRON BUNCH SHAPE

6.24. Of the devices described in Chapter V, the position finding scheme is not applicable to the cyclotron because of the distribution of the particles in space. The phase measuring scheme seems applicable at first sight. However, the frequency of the cyclotron (10 megacycles) is located at the poor end of the range. In the synchrotron, the variation of beam length is expected to be of interest in spite of the low definition of wave form achieved. Again, the cyclotron output can be assumed without measurement to be somewhat shorter than the 90 degree length expected to be reached at the end of the synchrotron cycle. Further, the whole output of the cyclotron will be shown to be just barely sufficient for direct collection and amplification. Thus, to investigate the particle motion in a cyclotron, special methods are required.

MEASUREMENT OF AMPLITUDES OF HARMONICS

6.25. The first measurement on the wave form of the cyclotron output was made by Dr. Wilson, formerly of this laboratory. He measured the amplitude of the first six harmonics by tuning a superheterodyne receiver to the six frequencies expected, one at a time. The converter was of a type believed to have constant response over the band. The amplitude of the nth harmonic of a simple rectangular pulse is

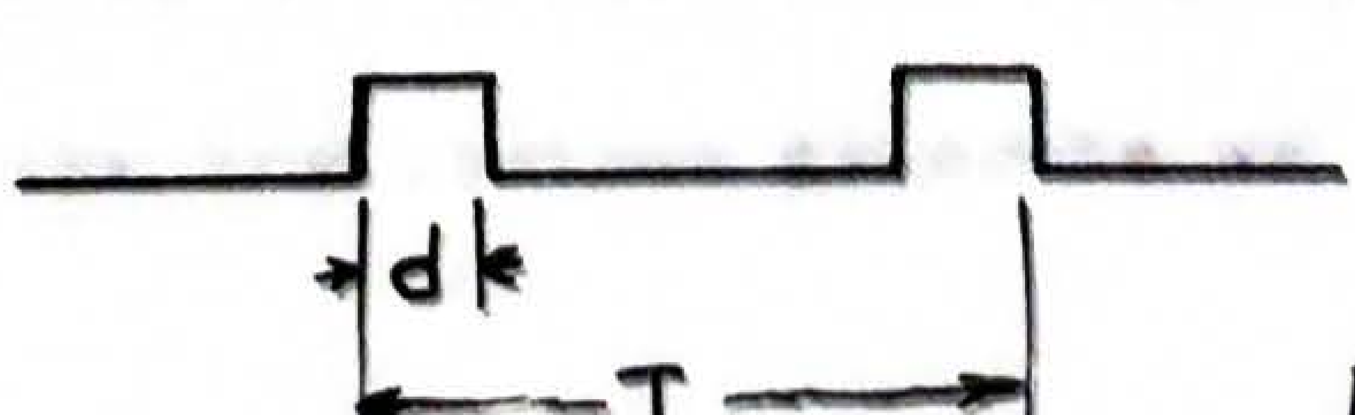


Fig 6.25

$$C_n = \frac{2Ad}{T}\left[\frac{\sin\frac{n\pi d}{T}}{\frac{n\pi d}{T}}\right] . \qquad (6.25.1)$$

The frequency sensitive term in brackets is the well-known function $\frac{\sin x}{x}$. For a 90° pulse, C_n is sero first when $n = 4$. For a 30° pulse, C_n has its first sero when $n = 12$, and the ratio of the amplitude of the sixth harmonic to that of the first is $\frac{2}{\pi}$. The results of Dr. Wilson's measurements showed that the first harmonics are all about the same amplitude, thus supporting the assumption that the cyclotron pulse width might be no longer than 30°. These measure-ments do not give a unique shape of wave form because the phase of the harmonics is not known.

POSSIBLE METHODS OF DIRECTLY MEASURING CYCLOTRON WAVE FORM

6.26. It is permissible to use the whole energy of the cyclotron output, 5 to 10 watts, in this measurement. Therefore one might suppose that a direct measurement of some sort would be possible. A single deflection of the protons by a rising electrostatic field could be made. A deflection of a few inches could be obtained by applying several hundred kilovolts to bars a foot or so long. A single linear rise of voltage to two hundred kilovolts could be produced by discharging a spark gap. The direct photographic record of the ions would then show several cycles of trace of variable intensity which would give the desired information. The high energy of the beam is, on the whole, a disadvantage in the previous measurement, since it effectively rules out a recurrent timed sweep. It might be possible to make the protons generate an electron beam

with a corresponding intensity. The protons might be directed against a secondary emitting surface, or be made to ionize a thin layer of gas, either of which would then provide electrons which could be used in an otherwise conventional cathode-ray tube structure. In Appendix F a simple method for sweeping a cathode-ray tube with a linear 10 megacycle time base locked to a 10 megacycle generator has been given. Direct amplification of the voltage developed by directing the beam into a small Faraday cage is also possible, but because of the high frequency, there seems to be no way of converting an appreciable fraction of the energy of the protons into an electrically useful form. It has, however, the advantage of flexibility when compared with the previous methods.

AMPLIFICATION AND DIRECT DISPLAY

6.27. When the beam is collected on a probe which is connected to earth through a 75 ohm resistor, the arrangement which was used in the harmonic measuring experiments, the energy developed by a steady beam is 107 db. below the beam energy. The noise power in the resistor, for 100 megacycle band width, is about 30 db. below this signal. A further loss of signal to noise ratio of more than 10 db. occurs when this signal is applied to the best tube available, since for this section of the frequency band a noise figure of better than 1000 ohms cannot be obtained. Voltage amplification can be obtained with a travelling wave amplifier in which the

outputs of a number of tubes are paralleled without paralleling their output capacities, although for the voltage gain required such amplifiers are very cumbersome. The most difficult problem in displaying a band of a hundred megacycles is applying the signal to the cathode ray tube. Tubes in which the capacity of the deflection system has been reduced to nearly the minimum capacity have not been available commercially. Indeed, since the early war years, side seal tubes are rarely seen. A travelling wave deflection system, in which the capacity of the deflector is broken up into parts and built into a transmission line, has been suggested. It is difficult, however, to design the tube with a satisfactory spacing. A cathode ray tube with single deflector pair, in which the capacity of the deflector system has been reduced to the capacity which is necessary (a few micromicrofarads), can be conveniently fed by a travelling wave amplifier using high frequency tubes. To obtain sufficient deflection requires that a considerable number of tubes be paralleled in the output stage. It may be shown that scaling down the cathode ray tube structure makes it easier to obtain the necessary definition, but again there is some difficulty in procuring a suitable tube.

6.28. When the poor signal-to-noise level of the voltage to be measured is considered in conjunction with the complexity of the high frequency circuit and the difficulty of obtaining a satisfactory cathode ray tube, it would seem that a more convenient method should be sought. This conclusion is supported

by the fact that, up to the present, little use has been made of direct display on a cathode ray tube for a hundred megacycle frequency range except when large voltages are available and amplification is not required.

STROBOSCOPIC WAVE FORM AND PHASE MEASUREMENT

6.29. The stroboscopic method of measurement appears to offer considerable simplification in equipment, since the band width may be reduced by a factor of about 10^7 and the frequency handled by most of the equipment is in the audio range. The stroboscopic measurements are made by repeatedly observing the voltage at a point in the cycle for a short fraction of the cycle. The wave form is built up by moving the observation point slowly over the cycle. The first such wave form measurement was made in 1870 on an alternator by connecting a voltmeter to it through a short rotating sector driven by the alternator, and a stationary brush of adjustable angle. In the measurement to be described, a crystal or diode, controlled by a suitable short pulse locked to the R.F. dee voltage plays the rôle of the brush and sector.

6.30. Because the voltage to be measured is so small, however, it would be impossible in a simple measurement to distinguish it from contact potentials in the gate element. Further, since the gating pulse is a few volts, and the signal a fraction of a millivolt, a constancy of 1 part in 100,000 would be necessary to allow the observation of the signal to a few per cent. Since the extra noise sources referred to

above are confined chiefly to the very low frequency region, the solution is to modulate the proton beam. Then, in the gate circuit, the only voltage distinguished by modulation is that corresponding to the stroboscopically observed point in the cyclotron beam wave form. This is a very desirable situation and makes the measurement possible. It is also a help in reducing noise picked up through unwanted coupling to other equipment. Modulation is merely a precaution against introducing spurious noise in the gate circuit, and the noise figure now falls to a value determined by the gate noise in the band of frequencies accepted. The final detector is of the phase-sensitive type, and the band width may conveniently be a few cycles if desired. A block diagram of the apparatus is shown below. The components listed on the diagram will be discussed in detail, proceeding from left to right.

PHASE SHIFTING DEVICE

6.31. The phase shifting device is merely a terminated line with a travelling probe to pick up a voltage from a variable point on the line. Since a concentric line using a plain centre conductor would have to be over 30 meters long to provide 360° phase shift, the centre conductor takes the form of a helix. When the centre rod in a concentric line is replaced by a helical coil of the same outer diameter, the effect may be approximated by considering the capacity per unit length unchanged, and the inductance per unit length increased in the ratio of solenoid to rod inductance. Thus

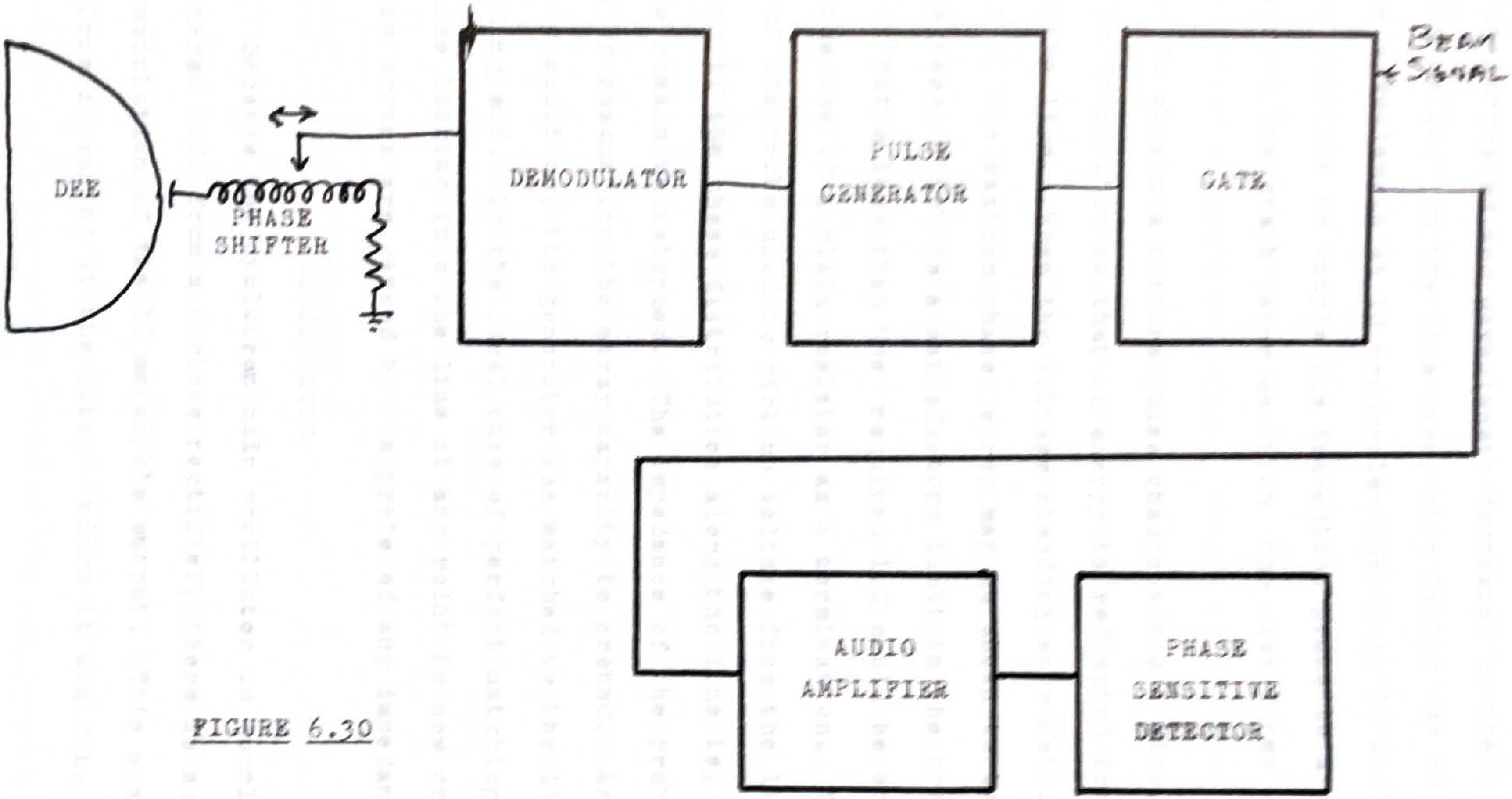

DEE
PHASE
SHIFTER
DEMODULATOR
PULSE
GENERATOR
GATE
BEAM
SIGNAL
AUDIO
AMPLIFIER
PHASE
SENSITIVE
DETECTOR

FIGURE 6.30

impedance is increased and wave-length decreased by the square root of this ratio. In the line used, this factor was adjusted to make the wave-length at 10 megacycles just over 10 inches. It was then possible to couple the travelling probe to a cursor which traversed the graph paper on which the wave forms were plotted.

6.32. To obtain a uniform phase change along a line, the line must be terminated so that no energy is reflected from the end of the line. When the voltage standing wave ratio is less than 1.2, the maximum phase error may be shown to be less than $\pm$ 5 degrees, which is a satisfactory limit in the present instance. A ratio less than the required 1.2 could be obtained by the use of a plain resistor as a termination. The impedance of the probe used to pick up voltage from the line must be high if the phase distribution along the line is, in general, to remain undisturbed. The impedance of the probe was raised by resonating the stray capacity to ground. As an additional precaution, the generator was matched to the line at the sending end. In the ideal case of perfect matching, the impedance looking into the line at any point is now constant, and no phase errors are caused by a probe of any impedance.

DEMODULATOR

6.33. Because the cyclotron main oscillator is supplied with unfiltered D.C. from a 6 phase rectifier, there is some 300 cycle modulation of the 10 megacycle output. This must be prevented from appearing at the gate, since it would be many

times the maximum expected signal, and even though the two voltages would be of a different frequency, separation would be very difficult. Since an acceptable gating pulse is difficult to produce, it is thought better to remove the modulation before the gating pulse circuit, rather than to attempt to clip the gating pulse. The latter procedure would very likely result in changing the pulse shape or phase at the modulating frequency.

6.34. An A.V.C. amplifier with a response sufficiently rapid to handle the modulating frequency was used. The design of A.V.C. amplifiers is discussed in Appendix C. The only new circumstance in the design of this device is the necessity for avoiding any more than a few degrees of variable phase shift during the modulation cycle. This is done by making the plate current in the tubes low and using low load resistors. For convenience in operation, this part of the circuit also supplies a constant R.F. voltage to the gating device. It is thus unnecessary to adjust for different dee voltages. The demodulation obtained reduces the 300 cycle voltage at the gate to about the same magnitude as the signal. This is sufficient to prevent its having an effect upon the phase sensitive detector. For future work at lower values of signal, it would be desirable to reduce the modulation still further.

GATING PULSE

6.35. The gating pulse has been generated by applying a large sine wave to the grid of an EF91 with self bias. The pulse so produced is about 40° wide at the base. It can be sharpened slightly by choosing a value of the coupling condenser

which allows the coupling circuit to relax slightly on the flat top. The wave forms measured for various values of coupling condensers are shown in Figure 6.35 (b). The wave form of the completed generator was that shown for 17 micromicrofarads. The wave forms were measured by the stroboscopic device described in section 6.29. A better generator would seem to be a chain of stages producing eventually a square wave with the rise time of the plate and grid circuit, or possibly of the plate circuit alone. A shorted line could be used to convert the square wave to a pulse. It seems reasonable to hope that a gate-open time of 10^{-9} seconds or 3.6 degrees at 10 megacycles could be obtained from a circuit using standard tubes. Specially designed cathode ray tubes have interesting properties as pulse producers but entail a sacrifice of convenience.

MODULATION

6.36. The proton beam must be modulated by a frequency which is distinctive, that is, which does not enter the gate circuit from any other source. 1000 cycles is a convenient choice, since it is well above the excess cathode noise region in most amplifiers. There are several methods of modulating the proton beam falling on the receiving plate, such as interposing a rotating toothed wheel or a vibrating reed between output aperture and plate. A contact breaker might also be used. The design of low noise contact materials has reached a high point of perfection. When the very best signal to noise ratio is not required, that is, for voltages more than a

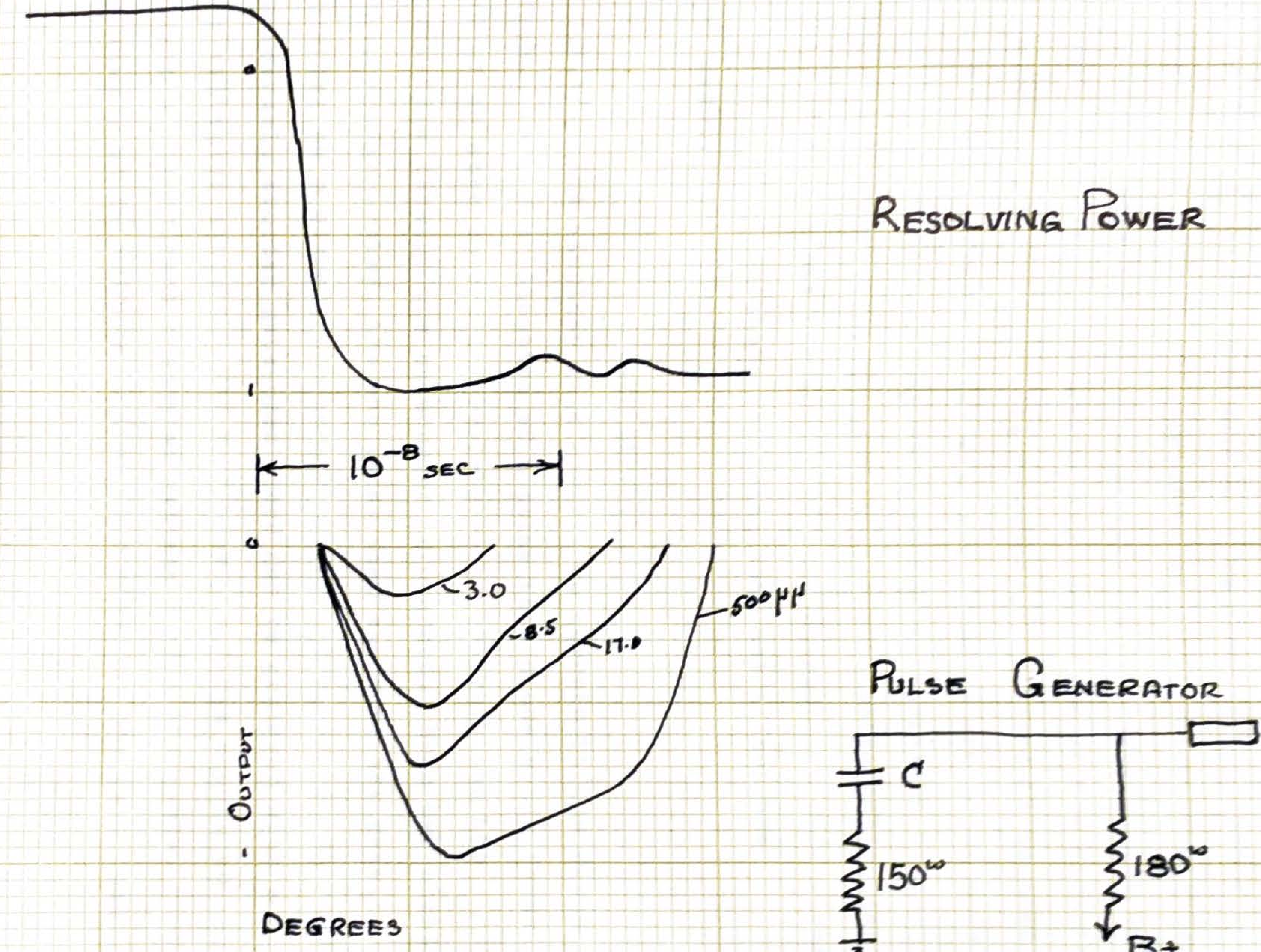
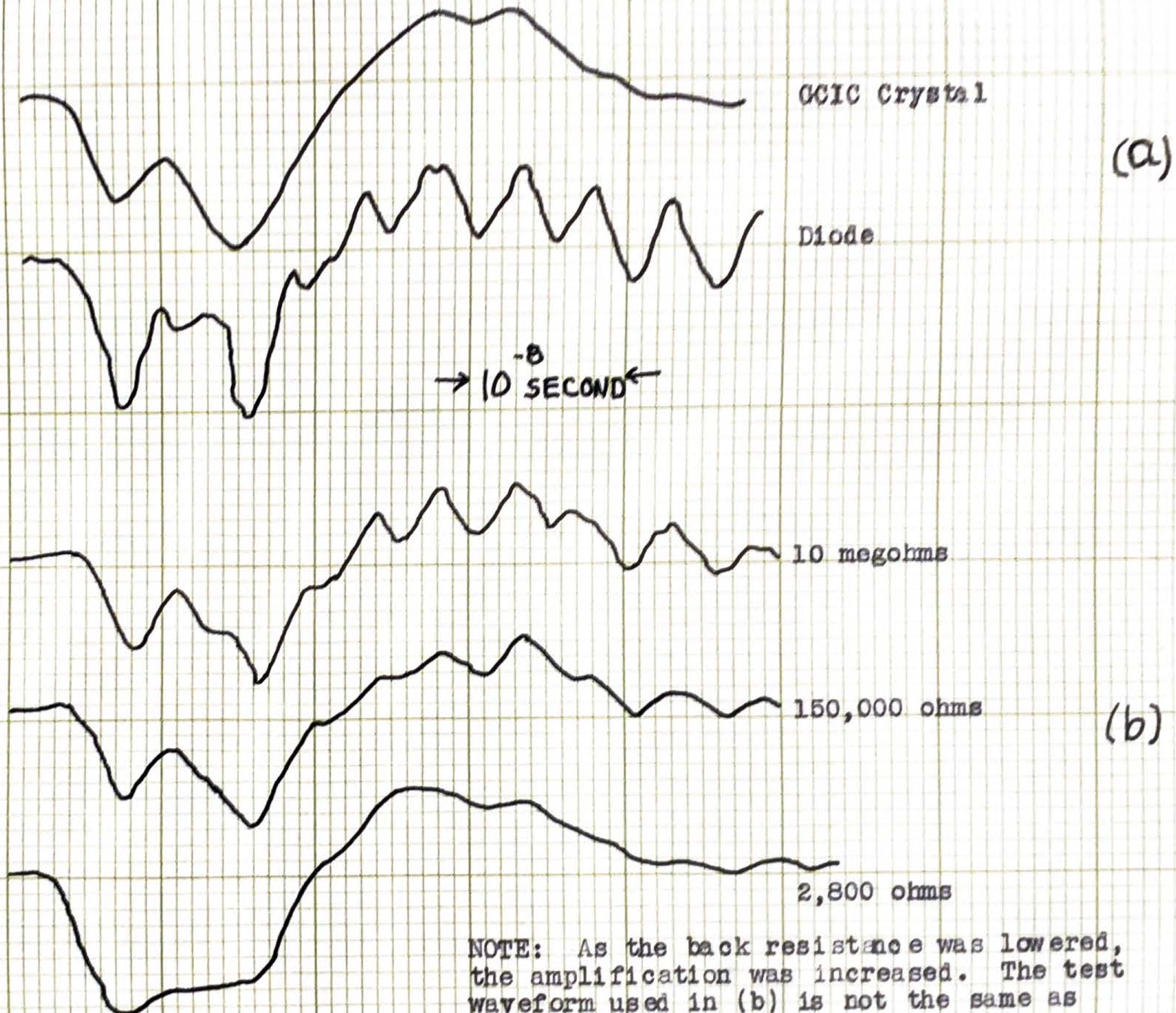

(a)
Resolving Power
10^{-8} sec
(b)
3.0
8.5
17.0
500 μμf
Output
Degrees
Pulse Generator
C
150 ω
180 ω
B+

Fig 6·35

millivolt or so, an electronic switch is the most convenient modulator. The design of a switch for another purpose has been discussed in Appendix D. The plate load resistor must be made very small, and the maximum gain will be about 1/2. Such a switch was used in design experiments and would be suitable for measuring voltages occurring in various parts of the cyclotron oscillator circuit. Fortunately, the ion source of the cyclotron can be modulated at a suitable frequency, and there is no need for other means of modulation when measurements are being made on the cyclotron beam at any point in the acceleration cycle. In addition to maintaining the best signal to noise ratio, ion source modulation has the advantage that unmodulated R.F. picked up due to lack of adequate shielding does not affect the measurement.

GATE CIRCUIT

6.37. The simplest gate circuit to compute is the diode switch. On the application of a pulse to a diode which forms part of an attenuator, its resistance is changed, and the attenuation through the attenuator is changed. When two similar pulses are generated, the one being used to drive the gate, the other as a signal, the length of one pulse may be estimated. Considerably shorter gate-open times may be achieved with a peak voltmeter circuit. The condenser is large enough to prevent a significant change of voltage during the cycle. The steady state voltage is then determined by the fact that



NOTE: As the back resistance was lowered, the amplification was increased. The test waveform used in (b) is not the same as that used in (a).

Figure 6.57.2

gate is changed. The signal generator of figure 6.37.2 (b) is not exactly the same as that in figure 6.37.2 (a). It may be seen that the definition decreases as the back resistance in the circuit is lowered. When the resistance is raised above several megohms, no improvement can be noted with further increase.

6.38. A complicated wave form is unsuitable for estimating the performance of the circuit on simple forms such as a short pulse. A periodic input may produce in the measured wave form variations which are much shorter than the open time of the gate. A single pulse cannot, however, be recorded as having a length shorter than the time the gate is open. The best wave form for testing would be a pulse known to be much shorter than the gate-open time. Another suitable wave form would be a step function. If the response of the gate as a function of time were $y = f(t)$, an ideal step function would be reproduced with a side

$$y = \int_0^t f(t)dt \ .$$

It is, of course, very difficult to get a sufficiently close approximation to a step function, and especially difficult to estimate how close the approximation really is. A simple squaring circuit, such as a circuit in which the voltage rise is limited by grid conduction appears to be the best at hand. It may be assumed that there is no structure to the input wave too fine to be measured and capable of making the recorded rise

shorter than the gate-open time. Thus the result obtained gives an upper limit to the gate-open time. The result obtained is shown in Figure 6.35 (a). This is taken as an indication that the gate-open time is shorter than 3×10^{-9}. The irregularities following the steep descent in figure 6.35 (a) are due to high frequency resonances in the test signal generator.

6.39. When the performance of the gate and pulse generator provides a gate-open time of the order of 3×10^{-9}, considerable care must be used in choosing and connecting the elements in the high frequency part of the circuit. A high frequency diode for concentric line mounting would evidently be an advantage, and a coaxial disc resistor should be used. Standing wave measuring gear at a few hundred megacycles would be helpful in setting up the circuit. Since high frequency elements were not available, reflections from the termination were moved outside the interesting range of phases by making the go and return path nearly a half wave-length. Reflections could have been removed at the expense of 6 db. signal loss by terminating both ends of the cable, had this seemed worthwhile. It seems unlikely that the departures of the impedances from ideal high frequency elements actually caused an appreciable reduction in the resolving power of the device, although a plane-element diode with reduced transit time would likely have improved it. If a better pulse generator were to be used, the high frequency design of the circuit would have to be improved. The result of varying the matching

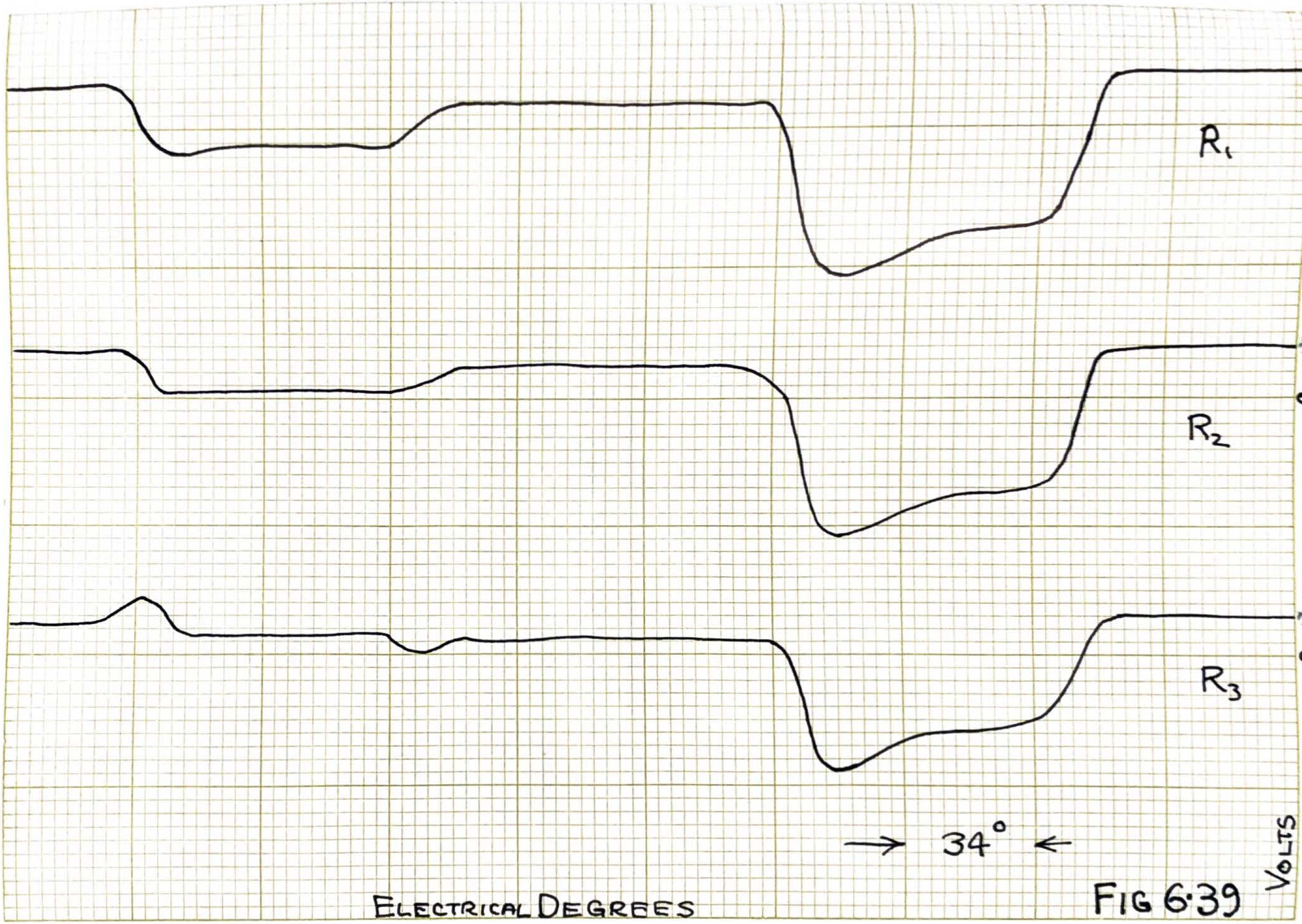


Fig 6·39

resistor using 1/4 watt carbon rod resistors is shown in Figure 6.39.

AUDIO AMPLIFIER

6.40. The audio amplifier is of standard design for low noise in the audio band. The amplifier noise is, however, somewhat smaller than the accidental audio modulation usually present on the gate pulse. A thermionic diode appears to make a negligible contribution, but if a crystal is used as a gate, the noise is increased many times because of a low-frequency excess noise mechanism.

PHASE SENSITIVE DETECTOR

6.41. A phase sensitive detector is required to exclude residual 300 cycle voltages at the gate, and to give a positive-negative indication when the signal voltage crosses the zero axis. Once such a detector is used, it is an easy matter to obtain band widths of a small fraction of a cycle if desired, since reduction of the band width is accomplished by slowing up the response to the direct current output of the device. A balanced pentode pair with the control voltage applied to the suppressors is used as a detector. Because phase sensitive detectors with electronic parts always give some D.C. output in response to an interfering signal of large enough amplitude, a broadly tuned circuit precedes the phase sensitive detector to assist the discrimination against large unwanted frequencies some distance from the operating frequency.

RESULTS OBTAINED IN THE MEASUREMENT OF BUNCH SHAPE IN THE CYCLOTRON

6.42. In the preceding sections the method of measurement of the wave form of the current in the external beam of the cyclotron has been explained. The wave forms so obtained are shown by plots such as that in Figure 6.42. The curves shown are the usual plot of current versus time with time horizontal and increasing toward the right. The current is, however, shown increasing downward. A number of current versus time curves for different operating conditions are shown, each plotted on a separate current axis. They are arranged in such a way that corresponding times for all the curves lie in a vertical line. In Figure 6.42, each curve corresponds to a different magnetic field, the curves being arranged in order of field strength. The voltage between dee and ground was 75 K.V. throughout.

6.43. The variation of output phase range with magnetic field, obtained by measurement, may now be compared with that derived under the assumptions of cases IV, V and VI. The width of the phase range changes very little with magnetic field. There is no sign of a maximum width near the resonance value of field. There is no sign of a common limit throughout most of the range and only a possibility of one at the extreme magnetic field. The centre of the bunch shifts to earlier phases as the field strength increases. The maximum phase range reached is just over 30°. Thus, instead of the extreme variation of phase range with sharply marked maximum near resonance, which is found in case IV, we have an almost

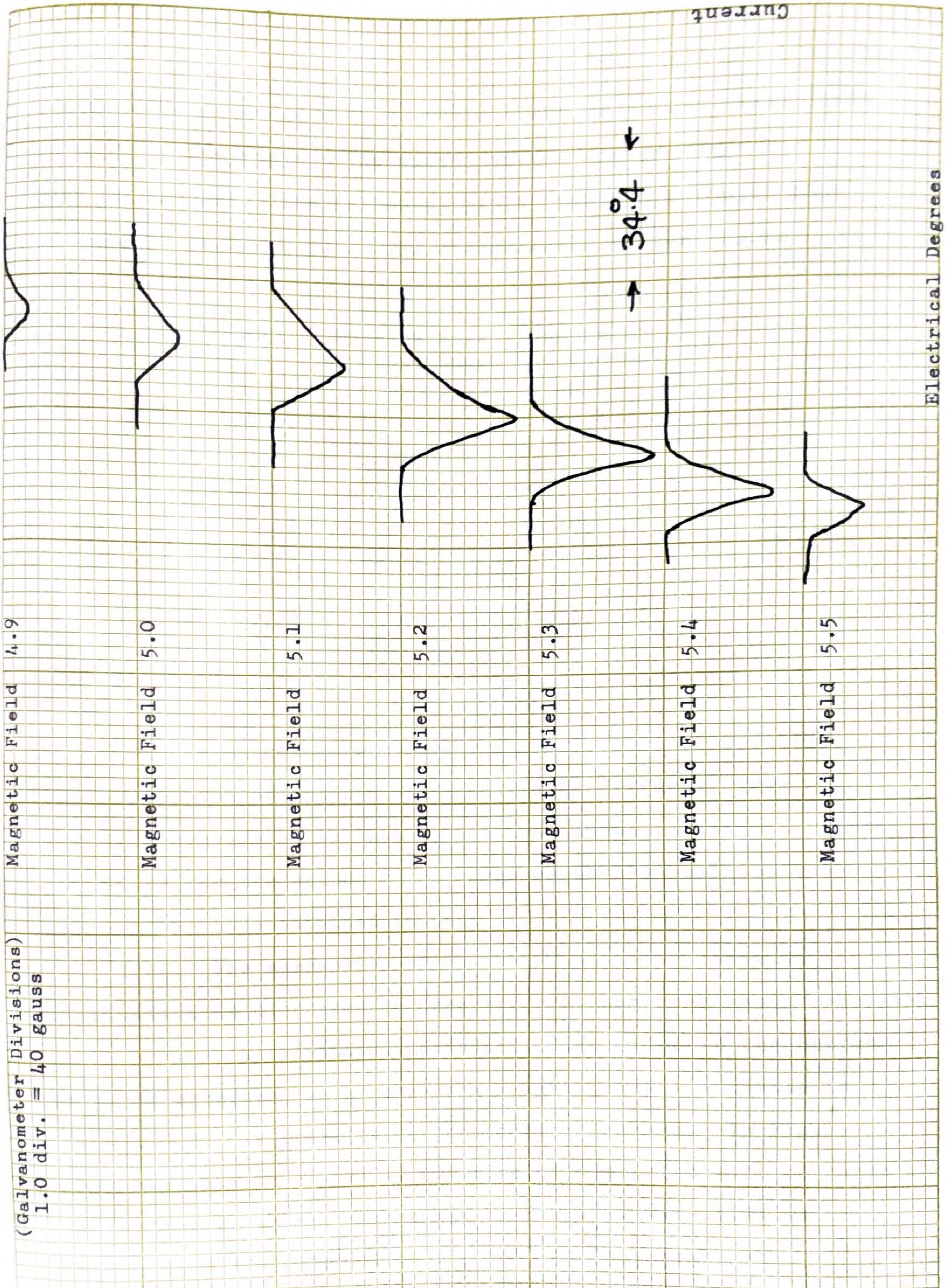

Current
Electrical Degrees
34.4°
(Galvanometer Divisions)
1.0 div. = 40 gauss
Magnetic Field 4.9
Magnetic Field 5.0
Magnetic Field 5.1
Magnetic Field 5.2
Magnetic Field 5.3
Magnetic Field 5.4
Magnetic Field 5.5

FIGURE 6.42

constant phase range with no maximum and no common phase limit in most of the range. The extent of the disagreement can be seen by comparing the following table (table 6.43) with that of case IV (table 6.19). The disagreement is seen to be

Δh	Outside Edge of Phase Range	Centre of Phase Range
0	72.4	60
-0.03	73.8	53.3
-0.06	60.0	41.2
-0.09	44.6	30.1
-0.12	30.8	19.4
-0.15	24	15.4
-0.165	20.5	12
-0.18	18.9	8.5

TABLE 6.43

sufficiently marked to disprove the assumptions of Case IV. The assumption of a lower bound on sin ϕ_0 different from -1, which is considered in case V, has been shown to lead to characteristics for the output phase range which are qualitatively similar to those obtained in Case IV.

6.44. The assumption of an upper and lower limit on input phase is discussed in Case VI. It has been shown (section 6.21) that an approximately constant phase range in the output is to be expected, with however a slight gradual increase in the width of the phase range toward later phases. In Figure 6.44, the rate of change of the edge of the pulse with magnetic field obtained by experiment can be compared with that which follows from the assumptions of Case VI. The graph also shows the variation of the center of the pulse. In comparing

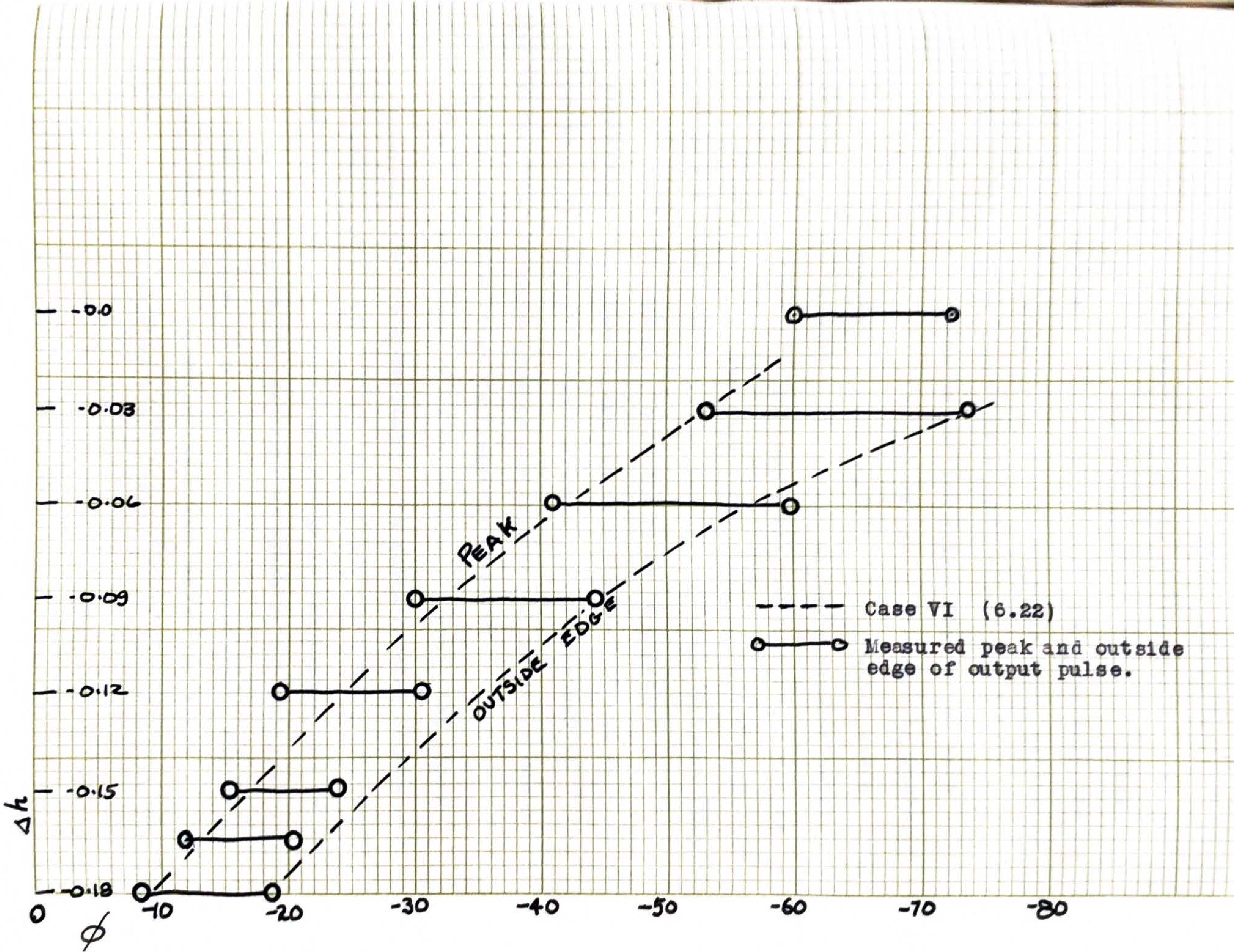

-0.0
-0.03
-0.06
-0.09
-0.12
-0.15
-0.18
Δh
0
-10
-20
-30
-40
-50
-60
-70
-80
φ
PEAK
OUTSIDE EDGE
Case VI (6.22)
Measured peak and outside edge of output pulse.

Figure 6.44

the two curves, it should be remembered that the cyclotron magnetic field and the frequency of the cyclotron R.F. accelerating voltage have not been measured in absolute terms. Thus the vertical position of the curves is arbitrary and only the change of phase with magnetic field is of interest. The choice of an absolute degree scale has a small effect on the fit obtained and is discussed in section 6.51. The output wave forms of figure 6.42 show the gradual small increase in width of the phase ranges toward more negative phases which was a characteristic of Case VI. The extent of the agreement between the experimental increase and that predicted in Case VI can be seen from Figure 6.44. The wave forms shown in Figure 6.42 also show a slight elongation of the lagging phase side as they move toward later phases. This also is to be expected from the assumptions of Case VI and not from those of IV and V.

6.45. Table 6.45 shows the output phase measured at different voltages. Resonance was obtained at as low a dee voltage as possible, that is, somewhat above 50 K.V. The characteristic features found experimentally are: the phase range quickly reaches a constant width which remains substantially unaltered as the voltage is raised; there is no sign of a fixed limit and no significant change in the centre of the phase range. Thus the assumptions of cases I, II, III are seen to be untenable. The extent of the disagreement may be seen from Table 6.45 in which the phase ranges for cases I, II, III are compared with those found experimentally.

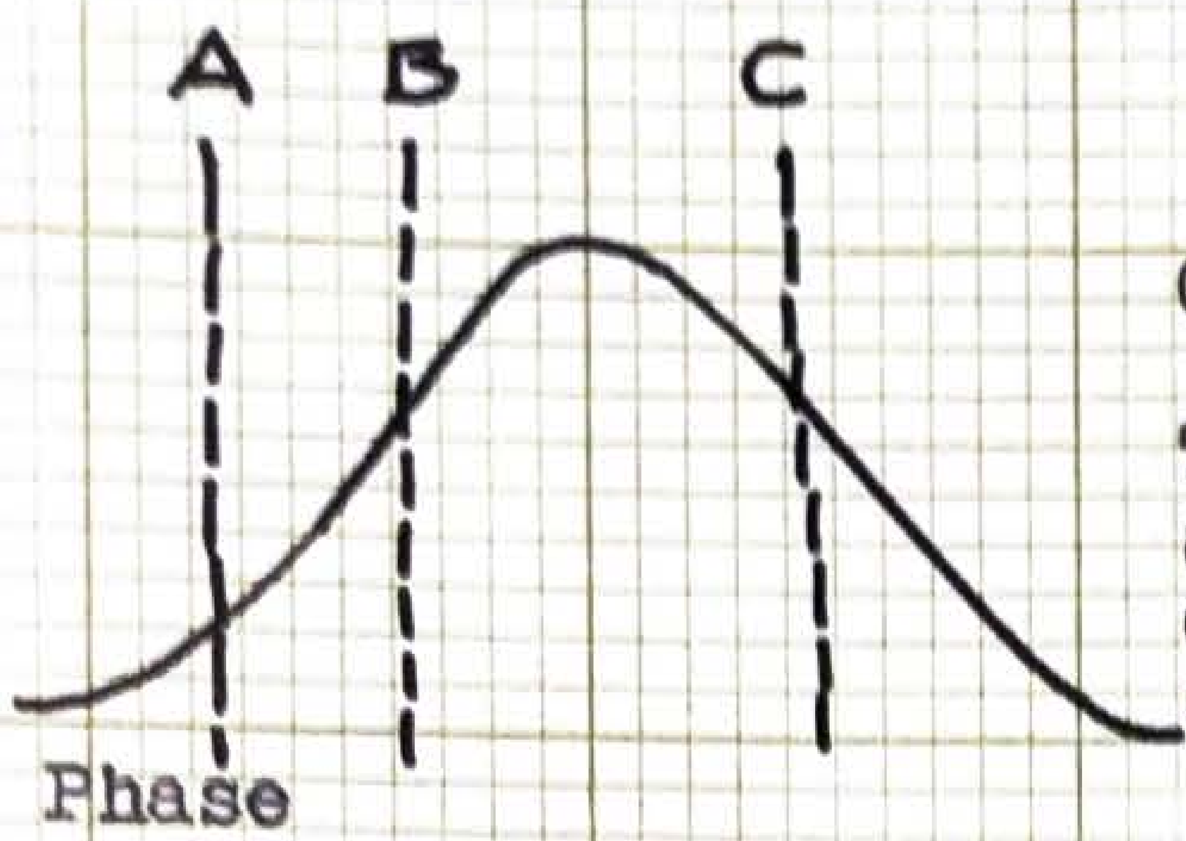
	Experimental Results		Case I		Case II		Case III h = -1.55		Case III h = -1.8		Case VII	
50			-64.2 to	-90	None		none		none		none	
55			-46.7	-90	-46.7 to	-60	none		none		-59 to	-49.8
60	-60.0 to	-43.0	-35.6	-90	-35.6	-60	none		-17.4 to	-19.5	-58.1	-38.0
65			-27.5	-90	-27.5	-60	-56.4 to	-60	-11.5	-22.7	-57.5	-29.5
70	-58.5	-31.3	-21.0	-90	-21.0	-60	-44.6	-60	- 6.6	-25.4	-56.8	-28.6
75			-15.5	-90	-15.5	-60	-36.0	-60	- 2.3	-27.9	-56.3	-28.1
80	-57.0	-35.0	-10.8	-90	-10.8	-60	-29.3	-60	1.5	-30.4	-55.9	-28.0
85			- 6.8	-90	- 6.8	-60	-23.7	-60	4.7	-32.0	-55.6	-27.7
90			- 3.3	-90	- 3.3	-60	-18.9	-60	7.7	-33.8	-55.2	-27.5
95			0	-90	0	-60	-14.7	-60	10.3	-35.4	-54.9	-27.3

TABLE 6.45

6.46. Table 6.45 also shows the type of variation expected in case VII where fixed upper and lower limits are assumed on the ion supply. This assumption gives the type of variation found in the experimental work. Again it should be noted that the absolute magnetic field is not known and that the variation of the limits of the output phase range with voltage is to be compared throughout the table, rather than the absolute position of the limits.

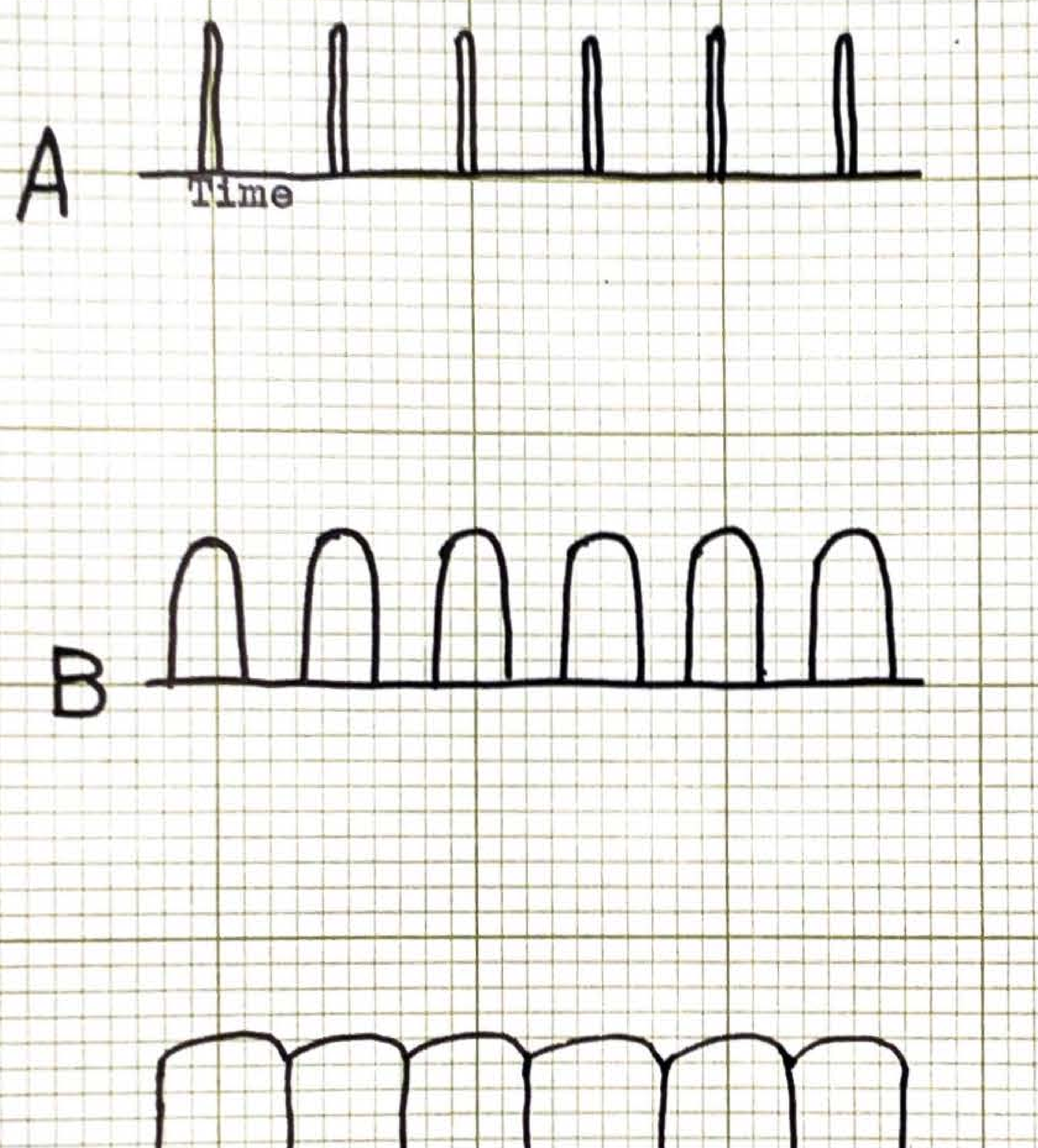
6.47. Further information about the variation of output phase range with voltage may be obtained when the cyclotron R.F. acceleration voltage is modulated by incomplete filtering of the power supply. When tuning is done at as low a voltage as can be used, it has been shown that the most lagging input and output phases are selected. Equation (6.23.2) shows that, over a range of voltages just above this, the range will be extended on the leading side by increasing the voltage, but that it will not be extended on the lagging side. Figure 6.47 shows the oscillogram obtained for beam current at a chosen phase versus time, during a cycle of mains supply. At early phases, such as A, no particles are obtained at all during most of the cycle. Particles are obtained only when the accelerating voltage reaches its highest value. (It will be seen that it does this six times per supply cycle.) At a later phase, such as B, particles are obtained during a larger fraction of the modulation cycle, but not for the lowest voltages reached. At phases such as C, very close to the lagging end of the phase range, particles are obtained almost

Figure 6.47



OUTPUT PULSE

The oscillograms A, B, C, show output current vs. time during one mains cycle at selected phases A, B, C.



continuously and with very little preference for the higher voltages. As the dee voltage is raised, the modulation nearly disappears at all phases. This would be true if, at the higher voltages, the limitation on output phase is due to the phase limitations on the ion supply rather than phase limitations imposed by the acceleration process.

6.48. Measurements on the variation of output phase range with magnetic field and with voltage have both been shown to require the assumption that the principal phase range limitation is due to the ion supply rather than to the acceleration process. The term "ion supply" is meant to include the ion source and the first revolution, that is, the part of the cyclotron where the acceleration process cannot be described by equations 6.5.3 and 6.5.4. It will be clear from the discussion of section 6.21 that $\Delta \sin\phi_o$, equation (6.21.3), is involved in these measurements and not the actual values of ϕ_o. On the other hand, the critical voltage, since it has to do with a single phase curve, does not depend upon $\Delta \sin\phi_o$ but does depend upon the actual value of the initial phase, or, more accurately, upon the lower limit of initial phase. The observed value of critical voltage is about 50 K.V. This suggests that the lower limit on the initial phase range is a negative angle rather than zero, or that limits such as B, figure 6.15, be chosen rather than A. It should be borne in mind, however, that the value of critical voltage depends upon such things as the

the calibration of the dee voltmeter and the exact distribution of magnetic field with radius. Thus the critical voltage measurement is not a good way of determining the actual value of input phase. The best method of deciding the input phase would be by a direct measurement inside the cyclotron. This measurement is discussed in section 6.61.

6.49. The width of the resonance curve depends upon both $\Delta\sin\phi_o$ and the actual value of $\phi_{o\ell}$, the lower limit of ϕ_o. The observed width of the resonance curve at 75 K.V. was found to be 0.3 per cent. This agrees with the value of $\Delta\sin\phi_o$ required by the magnetic field measurements and the assumption about $\phi_{o\ell}$ required by the voltage measurements. These assumptions are marked B on the resonance curves.

6.50. In the measurements described so far, only the simplest condition of operation of the cyclotron has been considered. This condition is obtained by placing a stop in a position across a diameter from the tip of the extractor plate. When this stop is not used, the appearance of the output changes radically and now has a double peak. The wave form of the output is then as shown in figure 6.50. The type of display used here is similar to that used in figure 6.42, that is, the current output is plotted against phase for a number of values of magnetic field. On closer inspection it may be seen that the left hand peak of the curve is very similar to that which is obtained with the simpler type of operation. That the left hand peak corresponds exactly to the one obtained

(Galvanometer Divisions)
1.0 div. = 40 gauss
Magnetic Field 4.9
Magnetic Field 5.0
Magnetic Field 5.1
Magnetic Field 5.2
Magnetic Field 5.3
Magnetic Field 5.4
Magnetic Field 5.5
34°.4
Current
Electrical Degrees

FIGURE 6.50

with the stop in position may be verified by comparing a number of curves taken with and without the stop, the operation being performed as quickly as possible. The correspondence is obtained over the whole range of magnetic field.

6.51. As the magnetic field is decreased with the stop out, a particular value of magnetic field is reached at which the right hand or second peak disappears. Since the whole right hand section disappears at once, the ions which produce this pulse would appear to be striking some obstacle inside the cyclotron as the orbits expand due to a decrease in magnetic field. The fact that no trace of a phase limit appears on the curve indicates that this happens before the limiting phase is reached. A movement of 7/8 inches along the radius is sufficient to move the stop from a position where it does not affect the right hand peak to the position where the right hand peak is completely removed. When the right hand peak is removed, the indicated beam current decreases and a corresponding amount of power (160 watts) is developed on the stop. An interesting characteristic of the right hand peak is that when the cyclotron is not operating steadily, apparently because of sparking taking place between dees and ground, the right hand curge is considerably more unsteady than the left hand one. A low frequency modulation of around frequency 7 or 8 cycles is also observed occasionally on this peak. It seems probable that the second peak is due to a second group of ions originating from a different part of the arc source or

accelerated along a different path from the other group. The fact that there is considerable similarity between the shapes of the two halves of the output pulse suggests the latter possibility rather than the former. The fact that the right half is consistently slightly longer than the left half would be expected from the fact that the right hand one has a later phase, section 6.21. It will be noticed that the phases of the right hand curve are considerably later than those of the left hand curve. Since it is impossible for ions to occupy a phase region where $\phi \nless -90^\circ$, it is assumed that this constitutes an inflexible limit. Under conditions of simple acceleration a considerable energy gain per turn would be required for extraction, setting an earlier phase limit than -90°, as discussed at the beginning of the chapter. It is possible, however, that the second group of ions is enabled to escape at phases near 90° by the motion acquired through precession. There are other reasons for believing that precession is especially likely in the second group of ions. A rough measurement of absolute phase supports the assumption that the phase limit reached by the second group of ions is near 90°, but due to insufficient care in terminating cables, it is felt that much weight should not be given to this measurement.

6.52. If the two parts of the curve correspond to two different groups of ions, it would seem very likely that one has at least slightly more energy than the other. Without setting up special equipment, the only way of distinguishing

between ions of different energy would be to vary the voltage on the deflector plate which decreases the curvature of the beam sufficiently to get it away from the magnetic field. Figure 6.52.1 is a set of curves showing the variation of the curve with deflector voltage in place of with magnetic field. In this figure the cyclotron is operated under the conditions for simple acceleration, that is, with the stop in place. It will be seen that the shape of the curve is unchanged as deflector voltage is varied. There is a slight decrease in maximum amplitude at each end of the range which presumably indicates that some ions have reached and passed the limiting path. Figure 6.52.2 shows the result of varying the deflector voltage when the cyclotron is operated without the stop and the complex curve is produced. It will be seen that the shape of the curve now changes when the deflector voltage is varied. This change takes the form of altering the relative heights of the two peaks. This may be due to a difference in the energy of the two groups of ions.

$$\frac{mv^2}{R} = \frac{Hev}{c} - eE \qquad (6.52.1)$$

H = magnetic field
v = velocity
R = radius

m = mass of the ion
e = charge of the ion
c = velocity of light.

Equation 6.52.1 shows that the radius of the deflected path depends on velocity. Thus the deflection system will act as a crude analyser. However, inspection of the shape of the curves over the range of deflector voltages available shows

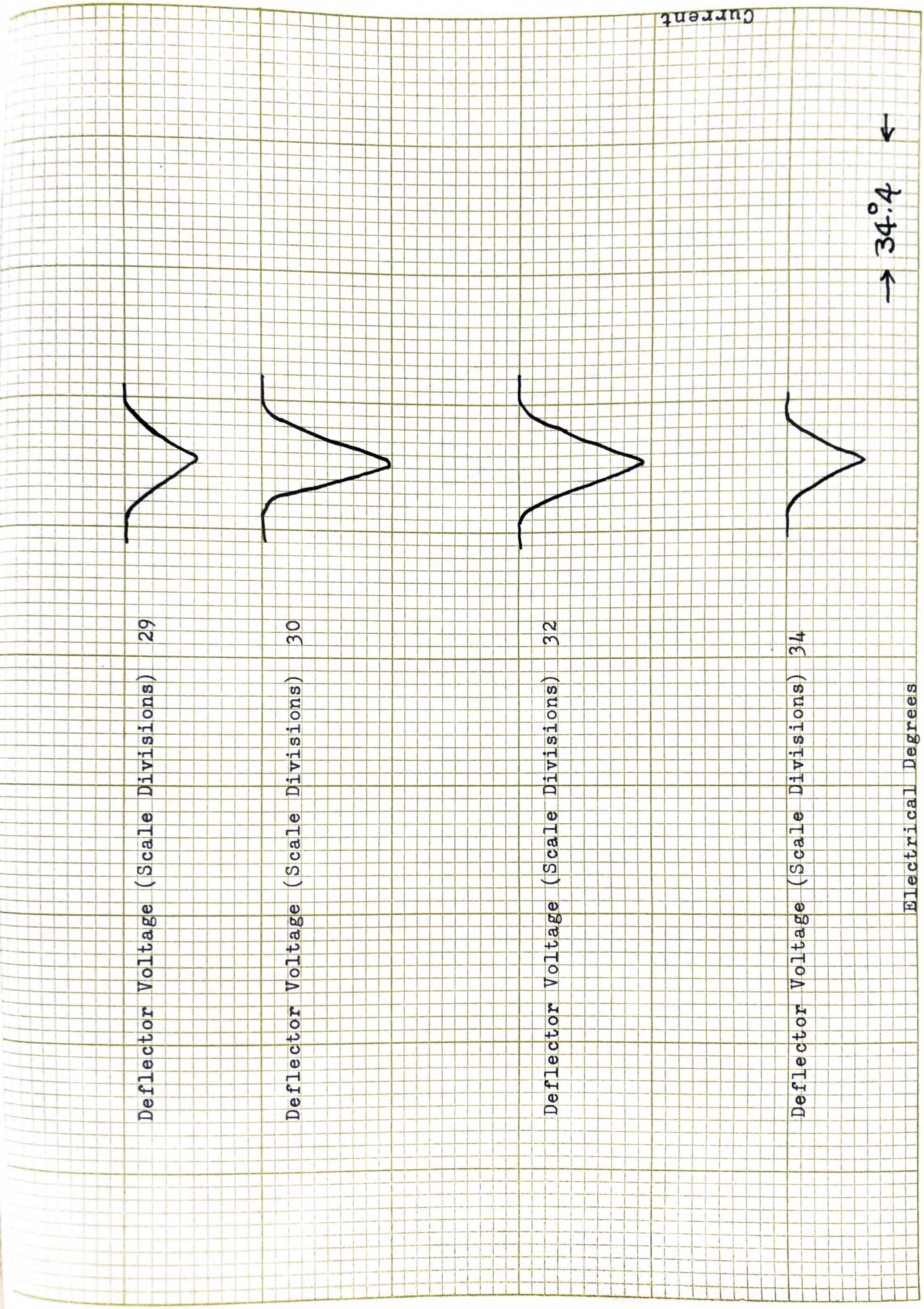

Current
Deflector Voltages (Scale Divisions) 29
Deflector Voltage (Scale Divisions) 30
Deflector Voltage (Scale Divisions) 32
Deflector Voltage (Scale Divisions) 34
Electrical Degrees
34.4°

FIGURE 6.52.1


Deflector Voltage (Scale Divisions) 30
Deflector Voltage (Scale Divisions) 31
Deflector Voltage (Scale Divisions) 32
Deflector Voltage (Scale Divisions) 34
Deflector Voltage (Scale Divisions) 37
Current
Electrical Degrees
34.4
32
34

FIGURE 6.52.2

that the change in the height of one curve is considerably greater than the change in the other. It is therefore possible that the two beams differ in position or direction in such a way that the extraction mechanism is different for the two beams.

6.53. Another attempt was made to estimate the energies of the two beams using available pieces of apparatus. It was thought that a movable slit could be varied in position so as to transmit selectively groups of beams having neighbouring values of radius. The equipment at hand proved inadequate to accomplish this.

6.54. If, in the experiment with the deflector voltage, the deflector is behaving as a simple analyser, then the left hand beam has the higher energy. This is the reverse of what one would expect from the fact that the right hand beam is the first one to be cut off as the stop is pushed in toward the centre of the cyclotron. It is also difficult to explain why the lower energy beam should have a later phase than the high energy one. This would be impossible if both started with the same phase and were accelerated in the same section of the magnet. It is possible that the right hand beam is precessing at a sufficiently rapid rate that it occupies, while passing the stop, a greater radial position than the higher energy beam. The situation can be clarified a great deal by a few simple experiments which are described at the end of this chapter, and it is probably unwise to speculate further until the result of these is known.

DISCUSSION OF THE MEASUREMENTS

6.55. In building the apparatus, it was necessary to keep the time required to construct the equipment to a minimum so that the writer might take as many measurements as possible in the time available. The parts of the circuits which would seem to respond to more development have been indicated in sections 6.29 to 6.41. The amount of shielding required was, unfortunately, underestimated. There is of course a great deal of radiation from the cyclotron radio frequency power source. The type of circuit was determined to minimize the trouble from this source. Another troublesome oscillator is the booster oscillator which is required to start the main oscillator because of the type of load presented to the main oscillator by small discharges in the cyclotron. This proved troublesome because it was not exactly the same frequency as the main oscillator and produced an annoying beat frequency in the audio range. In addition to these sources of interference, there was the trouble so often encountered when working in the vicinity of large high voltage power supplies. Especially when the amount of power produced by the supply is large, intermittent sparks may occur which do not interfere with the normal function of the supply but cause a great deal of trouble due to accompanying electromagnetic radiation from the sparks. In the interests of fundamental simplicity it would have been better to place the adjustable delay line in the cyclotron pit in order to avoid having long R.F. transmission lines from the pit to the control room and back. A

selsyn system could have been used to change the delay. Inspite of the troubles due to interference, consistent readings could be obtained. The phase sensitive detector was very valuable in allowing operation during periods of interference. Because the ion source was modulated there was very little chance of confusing signals from the beam with 10 megacycle voltages picked up from other sources.

6.56. The magnetic field was found the variable in the cyclotron most difficult to keep under adequate control. This was partly because the phase was so sensitive to the magnetic field and partly because the magnetic field of the cyclotron cannot be changed rapidly. There is also a tendency in the magnetic field stabilizing equipment for a small slow periodic variation. The curves had to be taken point by point. A cursor was arranged to traverse the graph paper and vary the delay at the same time. Since the delay was a linear function of the position of the pick-up on the artificial line, the points could be plotted on a linear degree scale by locating each point at the edge of the cursor. This was a great help in plotting quickly in order to get the whole curve plotted before conditions had changed. Each curve was traversed many times to see whether or not the conditions had remained sufficiently steady to make the curve reliable. When sufficient care was taken in having a second observer read the magnetic field during the plotting of the curve, it was found that the curves would repeat very well. Although the magnetic field cannot be read to an absolute accuracy adequate for a good

check, no significant change in the phase of the curve for a given magnetic field was observed during the entire time covered by the experiments.

6.57. The output of the cyclotron has thus been shown to consist at resonance of bunches of particles occupying about 20 electrical degrees or 5.5×10^{-9} seconds. This condition obtains only when a second group of particles is prevented from being accelerated by interposing a stop situated diametrically opposite the exit point. The nature of the second group of particles has not yet been determined. When the second group of particles is suppressed, the output retains approximately the same pulse width over a wide range of operating conditions. Theoretical considerations indicate that the output will narrow when the dee voltage is reduced sufficiently, but the output is then too unsteady to permit reliable measurements. This would not likely be a useful mode of operation. The resolving time of the apparatus is thought to be better than 10° or 3×10^{-9} seconds.

6.58. Although the magnetic field cannot be maintained as steady as could be desired, the variations are small enough with respect to the width of the resonance curve to permit accurately establishing the rate of change of the pulse phase with magnetic field. The variation of the phase range of the output with magnetic field and voltage can be established with sufficient accuracy to show that the length of the output pulses is primarily determined by the phase limits placed on the supply of ions at the center of the cyclotron. The mechanism is thought to be that suggested by Allwood. This contains several important implications. The output of small

cyclotrons should have the same bunch shape as that of large cyclotrons operating near the ~~frequency~~ energy limit when the ion supplies are similar. The bunch shape of the beam will be approximately the same from the centre of the cyclotron to the outside. This is of great importance when it is desired to use the pulsed nature of the beam in the measurement of short lived radioactivity and in delayed emission experiments. It would be useful then to be able to use the internal beam at various radii. In this connection it should be noted that a debunching effect occurs for angles near 90°. The opposite effect occurs when the ions are collected at a phase closer to zero than that at which they started. Another conclusion is that if one wished to make the pulses still shorter it would be necessary to design the ion source and its surroundings to produce a narrower range of phases.

AUTOMATIC TUNING

6.59. It has been shown that the phase of the output is approximately a linear function of the magnetic field. Further, the phase is very closely tied up with the mechanism of acceleration and is therefore a good measure of how close magnetic field is to the value which would give the best operation for the existing values of the other variables. It has been shown, for instance, that the optimum phase changes very little for a change in dee to ground voltage. During the experiments, the phase of the beam was found to be a very reliable measure of magnetic field. Thus the phase of the beam at one point is a good variable to use in an automatic

tuning circuit which might, for instance, change the magnetic field to keep the phase of the beam at a previously determined value. To be of the most use in the operation of the cyclotron, it would be desirable to find some way of measuring the phase the apparatus for which can be left connected during any of the operations for which the cyclotron is used. The most convenient position for a pick up point might therefore be inside the dee which does not contain the deflection system. While it might be possible to use induction from the beam without collecting it, this would almost certainly involve shielding difficulties. A more satisfactory system would probably be a horizontal pipe of small diameter projecting into the beam space far enough to catch a small fraction of the ions. A small collector would be placed inside this, and would receive ions through a hole which would be covered with thin foil. If tuning on the external beam only were required, it would be possible to use a tube through which the particles pass inducing a voltage.

6.60. For the continuous measurement of phase an electronic phase shifter of the reactance tube type could be used, to vary the phase of the gating pulse periodically at a suitable frequency, which could be quite high. A second gating circuit could be used to derive from the wave form so obtained a voltage proportional to phase. This could then be fed into the magnetic field stabilizer producing a small correction. No modulation of the ion source would then be required. Although it would likely be impossible to avoid a small amount of R. F. pick up from the dees, this would probably do no harm as the rate of change of the sine wave would be small compared to the rate of change of the voltage produced by the beam.

POSSIBLE FUTURE EXPERIMENTS

6.61. A number of measurements using the apparatus described in this chapter suggest themselves. It would be desirable to measure the phase of the beam at various radii by inserting a shielded probe along a diameter. By choosing the diameter between the two dees, it is likely that this could be done without interfering with the operation of the cyclotron inside the smallest radius reached by the probe. Water cooling of the outside body of the shield would be required but the ions actually collected need not produce much heat on the collector. If heating proved too serious a problem, the ion source might be modulated with a short duty cycle. The hole in the shield used to admit the ions to the collector could be shielded with thin foil or provided with a grid. The necessary sliding seal is already in position on the cyclotron. This measurement would give valuable direct information about the nature of the phase restrictions imposed by the ion supply. It would also fill in further details in the picture of the operation of the cyclotron, since the phases and bunch shapes at all radii could be measured directly. If it were desired to redesign the ion source and its surroundings to provide still shorter pulses, this measurement would provide the best way of assessing the improvement obtained. By providing a method for collecting the beam at various positions, it should be possible to determine the real nature of the two beams of ions which have been found in the output (section 6.51). It is hoped that this measurement will be carried out in this laboratory.

6.62. It would also be interesting to measure the output phase range after it has passed through the recently completed analyser magnet. An analysis of the beam described in 6.52 should also be made for the purpose of determining the energy difference between the two groups of particles.

APPENDIX A

NOTES ON THE DESIGN OF WIDE BAND AMPLIFIERS IN THE RANGE 100 K.C. TO 10 Mc.

A.1. Throughout the foregoing it is clear that amplification is required in many different situations. In most cases, the band of frequencies to be amplified is 250 K.C. to 10 megacycles. The writer spent some time, therefore, in an analysis of the various parts of the circuit for the purpose of reducing the size and complexity of the amplifier. It was found that most of the amplifiers required could be quite simple and small. This made the devices proposed above seem practicable in spite of the large number of wide-band amplifiers involved.

A.2. Wheeler's figure of merit for a high frequency amplifier tube is

$$\frac{G_m}{\sqrt{C_g + C_p}}$$

where G_m is the transconductance of the tube, C_g the grid to ground capacity and C_p the plate to ground capacity. This assumes that the grid and plate capacities are built into a single section artificial line. If simple coupling is used, the appropriate figure of merit is

$$\frac{G_m}{C_g + C_p}.$$

In the following table, the tube capacities alone are given, although a better figure would be one including socket and smallest realizable circuit capacity. Since the tube having the best figure of merit has also nearly the largest G_m, the other capacity figure would not likely alter the choice. The addition of unavoidable capacities would lower the figure of merit of a

tube such as the 6AK5 (because of its low capacities and low G_m) and raise the figure of merit of the EF91.

TUBE	A	B	C	D	E	F	G
EF91	2.65	<u>0.85</u>	1200	3.3K	7.6	135	100
EF54	2.32	0.7	700	10K	7.7	110	81
6AK5	1.93	0.75		39K	5.0	119	88
SP61	1.9	0.4		1.8K	8.5	63	47
6AC7	2.09	0.5	750	2.1K	9.0	80	60
EF50	1.51	0.5	1400	4.0K	6.5	80	60

Column A: Wheeler's figure of merit, $G_m/\sqrt{C_g+C_p}$

Column B: Ordinary figure of merit for simple grid-plate coupling, $G_m/(C_g+C_p)$

Column C: Noise resistance

Column D: Transit time and cathode inductance input loading, 50 megacycles

Column E: Transconductance, milliamperes per volt

Column F: Frequency for a gain of 1 into parasitic capacities

Column G: Frequency of column F corrected to include circuit capacities by using the ratio found for the EF91

TABLE A.2

A.3. COMPONENTS

A.3. The capacities measured for a number of EF91 tubes in the shield base socket but without the shield were slightly in excess of the value given by the manufacturer.

Plate	2.6	micromicrofarads
Grid	7.3	
Socket	2.0 (both sockets)	
1" wire	<u>0.25</u>	
Sum:	12.15	micromicrofarads

TABLE A.3.1

The cathode resistor used in this section was 180 ohms. The G_m using this resistor is 6.3, that is, somewhat lower than the

usual value of 7.6. The capacities of all interstage components were measured.

No.26 wire about 1" off chassis	0.1 micromicrofarads per centimeter
Small postage stamp condenser, narrow face on chassis	1.8 micromicrofarads
Same condenser 1" off chassis	0.7 micromicrofarads
Tubular and disc ceramic condensers about socket height	0.5 micromicrofarads
Resistors with about 1cm. of lead (measured with one end grounded)	
1/4 watt, large end cap	0.22 micromicrofarads
1/8 and 1/16 watt	0.12 micromicrofarads

TABLE A.3.2

The list of unavoidable capacities can thus be extended by adding to the above list of tube and socket and wire capacities (table A.3.1):

Blocking Condenser	0.55 micromicrofarads
Plate and Grid Resistor	0.25 micromicrofarads .

The minimum capacity for a properly wired stage is then about 13 micromicrofarads. The stages actually used in this section measured between 13 and 14 micromicrofarads.

FEEDBACK AND LAYOUT

A.4. Careful consideration of the conditions causing oscillation is prompted by the fact that new amplifier layouts sometimes oscillate. The majority of the couplings between components in a video amplifier involve resistances and capacities only. When very high frequencies are involved, coupling by wave-guide modes must be considered, but usually the high attenuation in launching and receiving the wave makes coupling

in this manner of no import. For no oscillation to occur, the feedback over any path must be such that the component fed back in phase is less than the signal required to produce it. In an amplifier of many stages, since the phase shift varies rapidly with frequency on the edges of the band, the absolute value only need be considered. When there are many stages, the net feedback must often be kept smaller than the relative constancy of gain required over the band, since the sign of the feedback may reverse inside the band. A value of 5 per cent feedback is taken as a representative upper limit.

CAPACITY BETWEEN THE FIRST GRID AND LAST PLATE

A.5. Where C is the capacity between the first grid and last plate, C_i is the input capacity, and G the overall voltage gain,

$$C = \frac{C_i}{20\ G} \qquad \text{(A.5.1)}$$

or $C = 5 \times 10^{-6}$ micromicrofarads for a gain of 10^5 and an input capacity of 10 micromicrofarads. Although this is a small value, the intercapacity may be kept below this value when tubes are wired in line on a metal chassis and small coupling condensers are used.

CAPACITY FROM ONE STAGE TO THE NEXT

A.6. This capacity produces the well known Miller effect, an increase in effective input capacity. Where G is the stage gain, the additional capacity loading C' produced by an interstage capacity of C is

$$C' = C(G + 1) \qquad \text{(A.6.1)}$$

CAPACITY FROM ONE STAGE TO THE NEXT BUT ONE

A.7. This coupling produces positive feedback for usual designs. When similar stages are wired in line, this coupling is small if the design is such as to produce a negligible value of Miller capacity.

HEATER COUPLING

A.8. The EF91 filament is a hairpin of coiled wire of about 20 ohms resistance (hot). It may be considered a pure resistance at the frequencies under consideration. Because of the close spacing of tubes, the connecting wire has negligible inductance and all filaments are essentially in parallel. The resultant resistance for 5 tubes is 4 ohms. The circuit is:

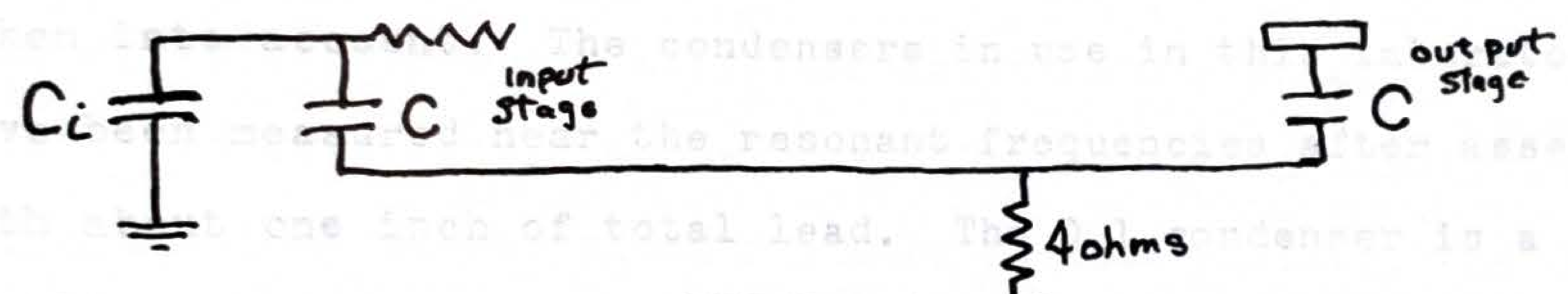


FIGURE A.8

The transmission increases with frequency at 6 db. per octave. It is

$$\frac{8\pi F C^2}{C_i} \qquad \text{(A.8.1)}$$

Feedback is greatest at the upper frequency end of the band in the neighbourhood of the point where the rate of fall off has reached 6 db. per octave. For an upper frequency limit of 12 megacycles, additional elements are required when the gain reaches about 10^5. Because of the low heater resistance, a choke isolating the first half of the amplifier is the most convenient solution.

PLATE SUPPLY LINE

A.9. The coupling due to a common plate supply is pG, where G is the gain from the plate of the first tube supplied to the plate of the last tube supplied, and p is the impedance of the by-pass condenser expressed as a fraction of the last tube plate load. If the worst phase conditions are assumed, the condition which must be met at all frequencies is

$$pG < 0.05 \quad \text{(A.9.1)}$$

If there are several sections, the condition must be applied to all the feedback loops. When considering the high frequency end of the band, it must be remembered that the inductance of connecting wire is about 1.3 ohms per inch at 10 megacycles. In addition, the inductance of the by-pass condensers must be taken into account. The condensers in use in this laboratory have been measured near the resonant frequencies after assembly with about one inch of total lead. The 0.1 condenser is a Dubilier, 350 volt condenser; the others are Aerovox "Postage Stamp" condensers.

BY-PASS CONDENSERS

Frequency / Condenser	100KC.	300KC.	1Mc.	3Mc.	10Mc.	30Mc.
0.1	16	5	1.25	0.05	1.25	(4)
0.01	160	50	16	4	0.15	(3)
0.005	320	100	32	10	3.2	0
0.001	1,600	500	160	50	16	5

TABLE A.9

The above table shows the approximate impedances of useful

by-pass condensers when assembled with nearly the shortest possible lead lengths.

A.10. None of the above considerations places any important restrictions on the tube spacing, therefore this may be made as close as mechanical convenience permits. A spacing of $1\frac{1}{4}$ inches is convenient when large by-pass condensers are used. When these are small, a 1 inch spacing is sufficient.

CIRCUITS

A.11. The R.C. circuit is to be preferred for its simplicity when the number of stages is small. In most of the foregoing we require an upper limit of at least 10 megacycles and a gain of 10^5. If stages giving a gain of 0.707 at 10 megacycles are used, 5 stages are required for a low frequency gain of 10^5. The whole amplifier has then a gain of $(0.707)^5 = 0.176$ at 10 megacycles. This is not a sufficiently uniform gain. As the plate resistor is made smaller, the flatness of each stage improves, but the number of stages increases. For the EF91 as operated, the gain varies with frequency as follows:

$$\text{Gain in db.} = 1.14 \times 10^{-3}\ n\ G^2F^2 \qquad \text{(A.11.1)}$$

G = the stage gain as a factor
F = the frequency in megacycles
n = the number of stages.

When the gain is fixed at 10^5, the optimum number of stages for maximum flatness is approximately 25. A fairly good approximation could be had with only 15 tubes, as the maximum is not sharp. These considerations show that for the gain desired the R.C. circuit is not suitable.

A.12. The feedback chain amplifier is well known in intermediate frequency amplifiers. An interesting improvement over the ordinary feedback chain amplifier consists in the addition of the shunt element C in the feedback path. As long as the

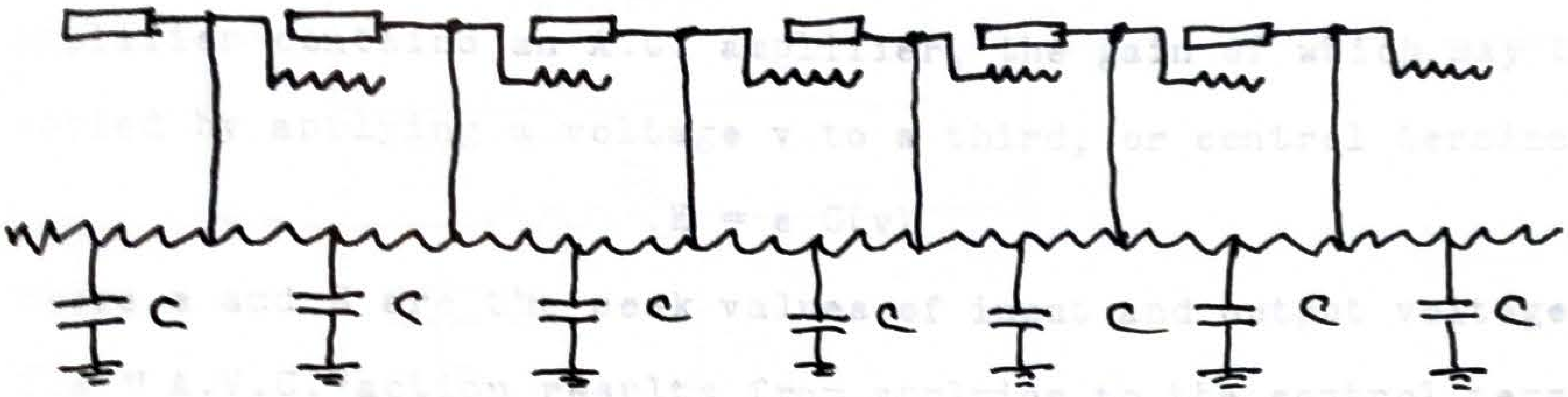


FIGURE A.12

feedback factor is large, an ideal amplifier with a feedback network having a complex characteristic β has a transmission characteristic β^{-1} in the region where the feedback factor remains large. β may be chosen equal to the transmission of the amplifier alone. Although ideal conditions cannot be closely met, a small improvement in flatness is of importance in a many stage amplifier. A gain of 2×10^4 was obtained in 5 stages with a drop of 0.2 db. at 8 megacycles, 0.9 db. at 9 megacycles and 1.8 db. at 10 megacycles.

A.13. Characteristics of standard forms of amplifier, such as the shunt peaking type, were also measured to provide data on a variety of circuits which might be used in the beam locating devices.

APPENDIX B

FEEDBACK LOOP IN THE CONTROL SIGNAL RATIO CIRCUIT

B.1. To see how the Nyquist condition applies to the complete feedback loop involving the control signal, we must get an equivalent circuit for the A.V.C. amplifier. An A.V.C. amplifier contains an A.C. amplifier, the gain of which may be varied by applying a voltage v to a third, or control terminal.

$$E = e\ G(v)$$

where e and E are the peak values of input and output voltages. The " A.V.C." action results from applying to the control terminal a voltage Δv equal to the departure of E from some standard value. If we require the same control action for any value of input, e, a restriction is placed on the nature of the function G(v). Since

$$\Delta V = \Delta E = e\ \Delta G(v)$$

$$\Delta V = E_o \frac{\Delta G(v)}{G(v)}$$

where E_o is the specified value of E. Thus the logarithm of the gain, or the gain expressed in decibels, must be proportional to the control voltage v. Since the tendency for the system to oscillate always sets an upper limit on the closeness of control obtainable, the result of not satisfying the above relationship is to reduce the average effectiveness of control.

B.2. The action of the A.V.C. amplifier may now be represented by an ordinary amplifier with feedback, where the feedback loop represents the reaction of the A.V.C. amplifier to modulations on the input.

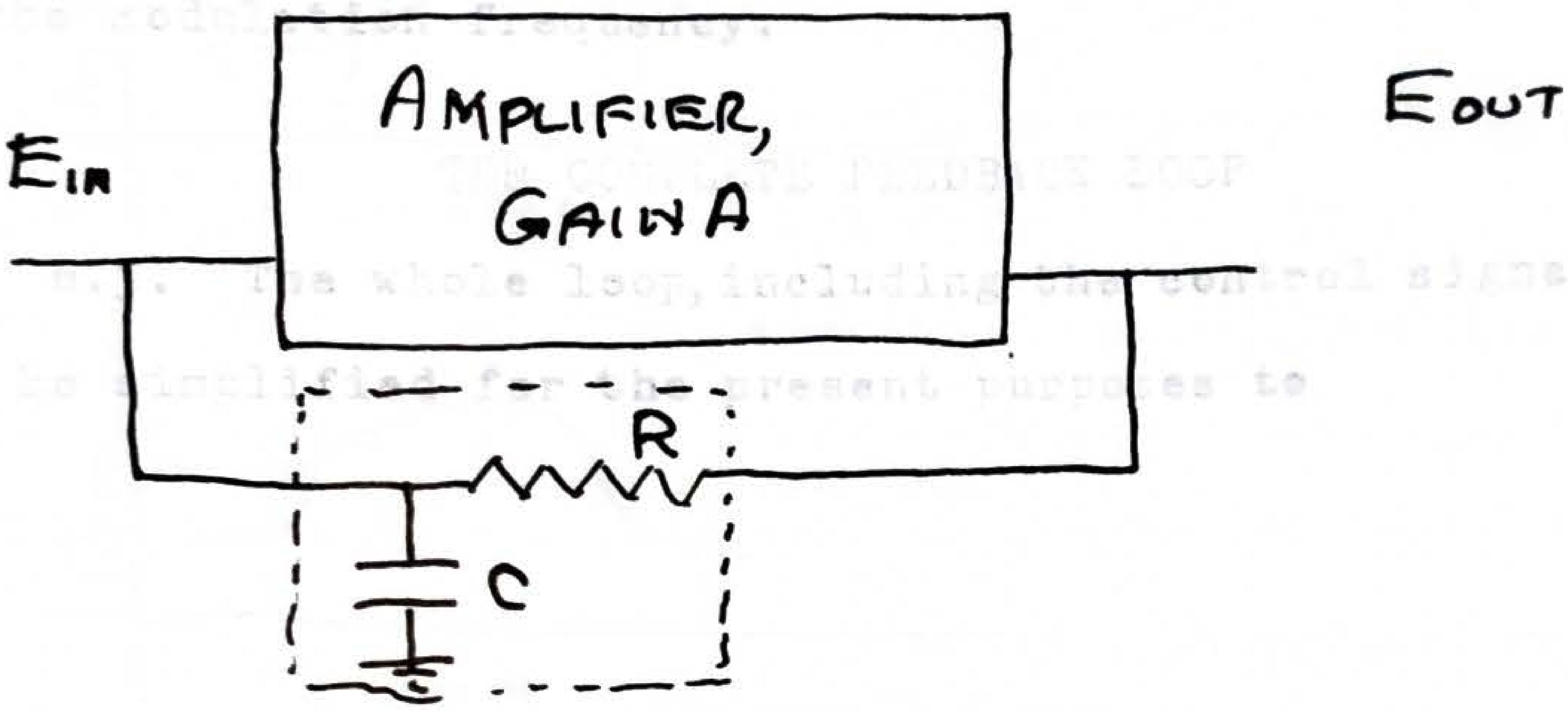


Figure B.2.1

A = amplifier gain = $\frac{\partial E}{\partial v}$ in the amplifier of section B.1 with the feedback loop open.

If the voltage from the feedback net and the input subtract, we get the well-known result

$$\frac{E_{out}}{E_{in}} = \frac{A}{1 + A\beta} \quad \text{where} \quad \beta = \frac{1}{1 + jR\omega C} \ . \qquad (B.2.1)$$

The transmission characteristic of the complete circuit is

$$\frac{E_{out}}{E_{in}} = \frac{A + jAR\omega C}{1 + A + jR\omega C} \qquad (B.2.2)$$

which is that of the network in Figure B.2.2. as long as A is

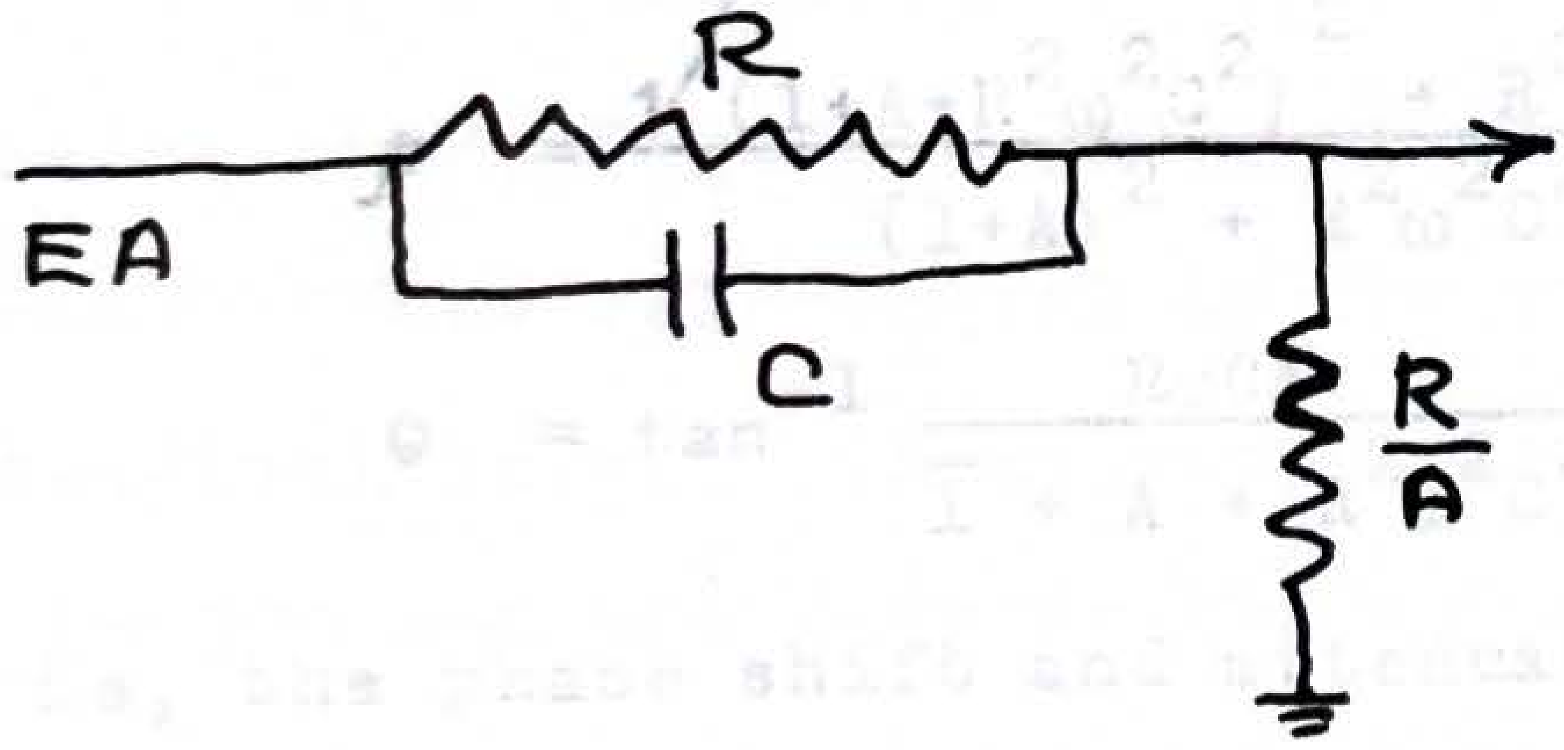


Figure B.2.2

considerably greater than one. This is true for any conditions of interest. The behaviour of the A.V.C. amplifier for small modulations is that of the equivalent circuit for applied voltages of the modulation frequency.

THE COMPLETE FEEDBACK LOOP

B.3. The whole loop, including the control signal generator, may be simplified for the present purposes to

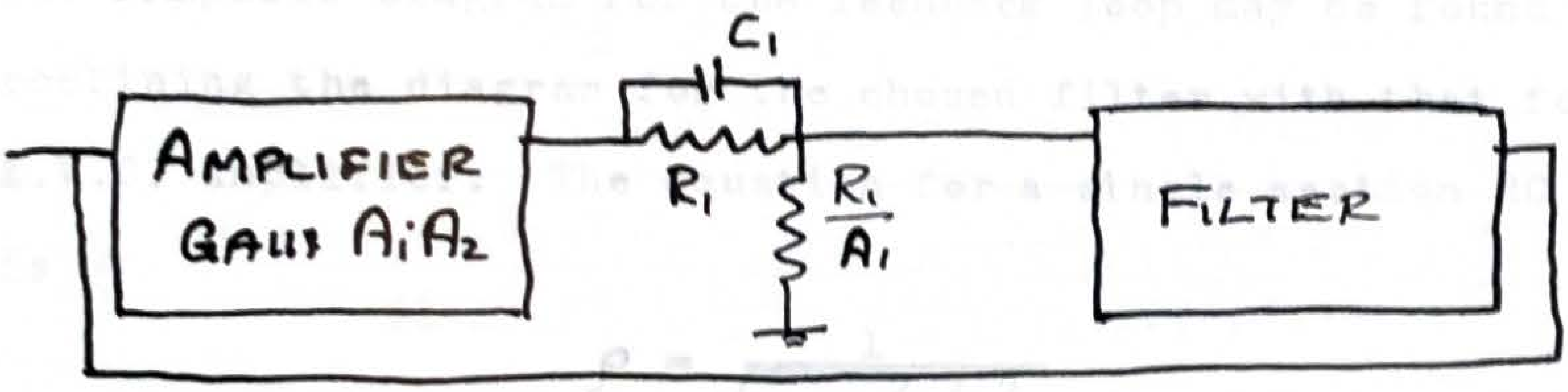


Figure B.3.1

The Nyquist diagram for the equivalent circuit of the A.V.C. amplifier may be obtained from the equations

$$\rho = \frac{\sqrt{(1+A+R^2\omega^2C^2)^2 + R^2\omega^2C^2A^2}}{(1+A)^2 + R^2\omega^2C^2}$$

$$\theta = \tan^{-1} \frac{R\omega CA}{1 + A + R^2\omega^2C^2} \qquad \text{(B.3.1)}$$

That is, the phase shift and attenuation characteristics as a function of frequency are approximately as shown in figure B.3.2.

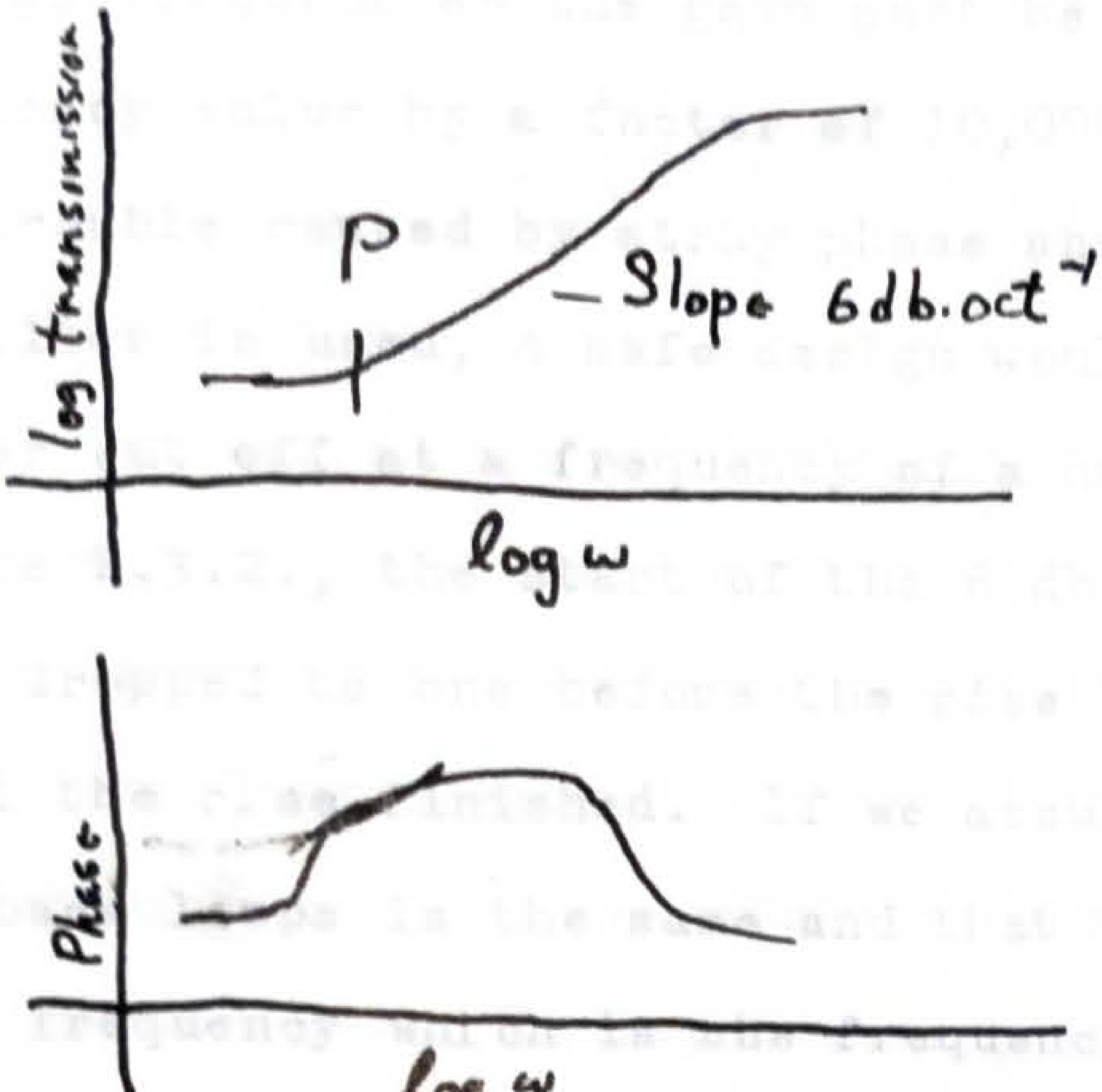


Figure B.3.2

The complete diagram for the feedback loop may be found by combining the diagram for the chosen filter with that for the A.V.C. amplifier. The equation for a single section RC filter is

$$\rho = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} \qquad (B.3.2)$$

$$\theta = \tan^{-1} -\omega RC .$$

The Nyquist diagram is thus the lower half of the circle

$$\rho = \cos \theta .$$

B.4. The chief difficulty in closing the feedback loop is the 6 db. per octave rise of gain encountered to the right of the point P, figure B.3.2., which continues until the gain has reached the value equal to the product of the gains in the two feedback loops. Thus, suppose the gain around the complete loop is 100 and the gain in the A.V.C. amplifier is also 100.

At high frequencies the gain must be reduced below the low frequency value by a factor of 10,000. This high gain exaggerates the trouble caused by stray phase shifts. If a single section RC filter is used, a safe design would result from starting the filter cut off at a frequency of a hundredth of that at P, figure B.3.2., the start of the 6 db. rise. The gain would then have dropped to one before the rise began and would remain constant until the rise finished. If we assume that the gain in both feedback loops is the same and that the filter cut off is started at a frequency which is the frequency at P divided by the gain in one loop,

$$R_2C_2 = AR_1C_1 \quad (= ARC) .$$

The Nyquist diagram for this situation is given by

$$\rho = \frac{\sqrt{\left(1+A+(1+A^2)R^2C^2\omega^2\right)^2 + A^2R^2C^2\omega^2(R^2C^2\omega^2 + A)^2}}{(1+A-AR^2C^2\omega^2)^2 + R^2C^2\omega^2(1+A+A^2)^2}$$

$$\theta = \tan^{-1} - \frac{ARC\omega(A + R^2C^2\omega^2)}{1 + A + (1+A^2)R^2C^2\omega^2} .$$

It can also be obtained by combining the two diagrams given in section B.3. It is seen that the Nyquist diagram lies in one quadrant and that the design is not an economical one.

B.5. An experiment was carried out with the complete loop as described above, the gains in both loops being 100. The safe design described above was found to be stable. However, Oscillation began before the cut-off for the single section filter had been moved up an octave. This type of circuit is thus fundamentally difficult to close. This constitutes a further reason for preferring the alternative circuit described in section 5.33.

APPENDIX C

A.V.C. AMPLIFIER FOR THE SWITCH RATIO CIRCUIT

C.1. In designing the A.V.C. amplifier one must avoid rectification in the grid circuit followed by a slow change in gain. A sharp low frequency cutoff is also desirable, since the switch transients have harmonics which decrease as the order decreases. It is not difficult to achieve a low frequency cutoff while retaining a high resistance in the control circuit. It is, however, very difficult to get a good anti-blocking circuit which lends itself to A.V.C. on the grid. Suppressor control, although requiring a much larger control voltage, is very desirable because of the isolation of the circuits. Because of the low suppressor to plate gain, R.F. feedback is not serious, provided the amplifier is operated at the highest possible level. The likelihood of R.F. feedback depends upon the ratio of control action to direct gain. This ratio is evidently highest at high levels. When the number of stages is large and the maximum gain is of the order of 2 to 3 times, the phase reversal from stage to stage helps to reduce the R.F. feedback. For the same overall gain, a large number of stages is preferable to a small one for the additional reason that the control factor is greater for the same direct gain. The fixed factors determining the seriousness of the R.F. feedback problem are the stabilization required and the speed of response required in the A.V.C. circuit

in relation to the low frequency cut-off of the amplifier. Both of these are unfavourable in this case, and in addition to using a large number of stages the minimum gain from the input of the A.V.C. amplifier must be used.

C.2. The minimum gain is determined by the magnitude of the signal which may be obtained before distortion exceeds the permissible amount. Distortion was measured by plotting output against input including suppressor voltage as a parameter. Roughly speaking, the maximum undistorted output varies directly as the gain, because of the current division between plate and screen. The distortion situation is somewhat better than in the case of grid control, because of the sharp cut-off of the EF 91. Since the distortion increases with decreasing gain, the limiting values are the high level ones (lowest gain). The highest average level is obtained when the gain over the A.V.C. amplifier is allowed to drop to a minimum of 1.0, as the following argument shows. If the minimum gain G were greater than 1, the critical level would be occurring in the final stage (at least overall gain,) and the input level would be 1/G of this. If the minimum gain G were less than one, the critical level would occur in the first stage and the subsequent loss in the amplifier would have to be made up in an uncontrolled amplifier. The last controlled stage would thus be at a level G times the critical level. Thus in both cases the level at some point in the amplifier drops below that which could be maintained at a minimum gain of 1.

C.3. Another consideration affecting the maximum level which may be used is the section of the suppressor characteristic chosen. For maximum level, the control should start at zero suppressor voltage and extend as far in the negative direction as is necessary to get the required range of gain. This makes it extremely difficult to get a logarithmic relation between gain and suppressor voltage. As may be seen from the published suppressor curves, the best approximation to a logarithmic relation is obtained in the extreme negative part. It is interesting to note that as the number of tubes is increased the overall characteristic approaches an exponential one, as long as the individual characteristics are continuous. This is because the operation of combining the gains (expressed in decibels) of individual tubes is one of addition. Where x represents the grid bias, and the gain is referred to the gain at x = 0:

$$\text{Gain of 1 tube in db.} = f(x) = xf'(0) + \frac{x^2}{2!}f''(0) + \ldots \quad (C.3.1)$$

$$\text{Gain of n tubes in db.} = nf(x)$$

$$= nxf'(0) + \frac{nx^2}{2!}f''(0) + \ldots \quad (C.3.2)$$

An ideal logarithmic relation (that is, linear in decibels) is:

$$G = nxf'(0). \quad (C.3.3)$$

For a gain of G = A, x has the value

$$\frac{A}{nf'(0)} \quad . \quad (C.3.4)$$

For an n tube amplifier, the gain is then

$$G = A + \frac{1}{n}\left[\frac{A}{f'(0)}\right]^2 \frac{f''(0)}{2!} + \ldots \quad . \quad (C.3.5)$$

The error terms are seen to decrease as the number of tubes, n,

increases. The practical limitation to the number of tubes used is that the dependence of the slope on other parameters, such as supply voltages, is proportional to the number of tubes.

C.4. Several points about suppressor control must be borne in mind. The plate resistance of high frequency pentodes, such as the EF 91 normally operated, is high at all frequencies where transit time and cathode lead inductance may be ignored. When the suppressor is made sufficiently negative to cut off the plate current appreciably, however, the plate resistance falls because the plate potential is capable of affecting the diversion of current to the screen. Because the stage gain considered is so low, the lowest values of plate resistance reached are well above the plate load.

C.5. Because the cathode current remains substantially constant, the screen dissipation may rise above the permissible value. This is a further reason for preferring low suppressor voltages. The screen voltage may be reduced, since the maximum transconductance is not required. However, this also lowers the maximum permissible level set by distortion.

C.6. When a large number of tubes is used with cathode bias, it is sometimes found that the maximum gain does not occur at zero suppressor volts. This seems to be due to the fact that the cathode current is not exactly constant as suppressor voltage changes, and the decreasing grid bias may have a stronger effect on the gain than the suppressor voltage. The effect can be prevented by the correct choice of cathode resistor.

C.7. While the suppressor impedance can be counted on

to be over 50 megohms for low negative biases, it has been found to drop noticeably at voltages in excess of 40 volts negative. The reason for this is not known. Fortunately, the effect is in the direction of improving the linearity (in db.) of the control curve. It is not sufficiently large that one need worry about changes of the effect from tube to tube.

C.8. The well known secondary emission effect which gives the suppressor a tendency to go positive, if driven far into the positive region, is not troublesome here, partly because the suppressor is never driven positive, and partly because some tubes do not show the effect and these tend to hold the other tubes.

C.9. The considerations given so far are all in favour of a large number of tubes. The gain range which must be covered is 1000 times, or 60 db. From a consideration of the control curves, a value of 7 has been chosen for the number of tubes. The stage gain is thus 9 db., or about 3 times. The load resistor and coupling constants are chosen to give this gain with a cut-off at 1 megacycle. The simplicity and good blocking characteristics made possible by the use of suppressor control are evident from the circuit diagram, figure C.9. The tubes may be mounted with 3/4 inch spacing. The high frequency design is no problem because of the low stage gain and plate impedance. Inverse feedback cannot be used because of the variable gain, but inductance compensation is quite satisfactory. As pointed out in section 5.33, the amplifier need not be particularly flat as it handles both signals.

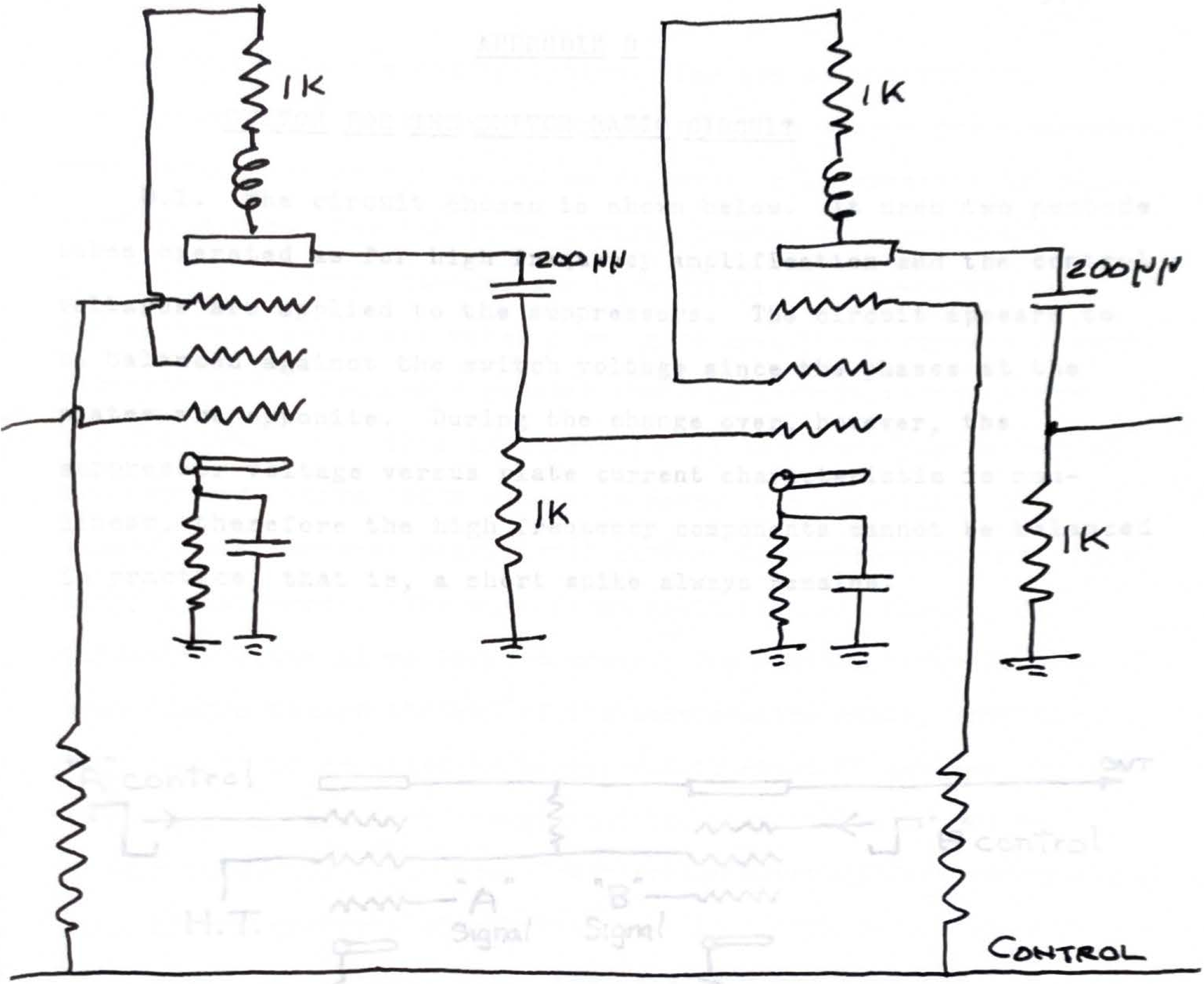


Figure C.9

APPENDIX D

SWITCH FOR THE SWITCH RATIO CIRCUIT

D.1. The circuit chosen is shown below. It uses two pentode tubes operated as for high frequency amplification and the control voltages are applied to the suppressors. The circuit appears to be balanced against the switch voltage since the phases at the plates are opposite. During the change over, however, the suppressor voltage versus plate current characteristic is non-linear, therefore the high frequency components cannot be balanced in practice, that is, a short spike always remains.

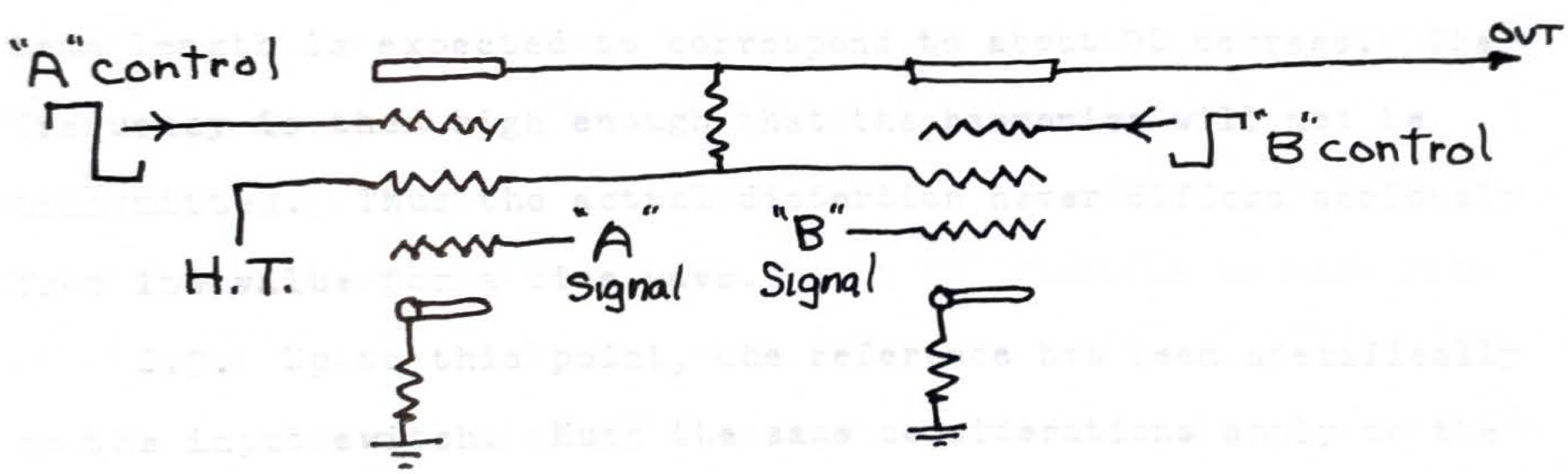


Figure D.1

A consideration of the accuracy required shows that it is not practicable to achieve a balance by the use of specially shaped switch wave forms.

D.2. To determine the maximum permissible signal level, the familiar linearity measurement, in which actual signal is plotted against signal extrapolated from zero level, was made

on the switch in the "on" position. For the output switch, the deviation from linearity must be measured with a peak measurement, since the output switch is followed by an essentially peak-reading diode. For the input switch, because of phase shift and high frequency cut-off, the wave form is not preserved and an average-reading meter gives more nearly the correct answer. The peak error is always greater than the average error, since the distortion consists of a fairly well localised flattening of the top and bottom peaks of a sine wave. The safe limit is about 3 milliamperes peak signal current (over the range of plate loads considered). The wave forms dealt with are always, fortunately, quite close to sine waves. The maximum harmonic content occurs toward the end of the accelerating cycle, when the beam length is expected to correspond to about 90 degrees. The frequency is then high enough that the harmonics will not be transmitted. Thus the actual distortion never differs seriously from its value for a sine wave.

D.3. Up to this point, the reference has been specifically to the input switch. Much the same considerations apply to the output switch, except that the range of levels encountered here is very small compared to those at the input switch. The circuit is, by nature, unbalanced, but this makes little difference, as has been shown (section D.1). The level at the switch is made as high as convenient to minimize the effect of local switching transients.

APPENDIX E

A.V.C. FILTER FOR THE SWITCH RATIO CIRCUIT

E.1. The A.V.C. filter allows the A.V.C. action to take place on the sum of the two outputs without distinguishing between them. If the sum were constant, or changed very slowly, a long-time-constant filter would be satisfactory. It is desired, however, to be able to accomodate intensity changes of as great a speed as the circuit design can be made to cover. When the voltages E_A and E_B are unequal, a rectangular wave in phase with the switching waves is applied to the right hand end of the A.V.C. filter. The transmitted component modulates the signal at the switch frequency introducing a spurious difference between different parts of the switch cycle. This is a systematic error in that it remains constant for any given ratio of E_A to E_B (as long as the characteristic of the A.V.C. amplifier is logarithmic). It is desirable to make this correction fairly small, however,

E.2. For a rapid build-up accompanied by a high attenuation of high frequencies, it is desirable to make the cut-off as rapid as possible. Since the phase shift depends upon the rate of change of attenuation, the Nyquist condition, which states that the loop gain must be less than one when the phase shift reaches 180°, limits the rate at which the loop gain may be reduced. Although, as Bode[31] has shown, the best design

(constant phase margin) results from increasing the slope of the cut-off characteristic near the edge of the band, in these experiments only simple RC filters were used.

APPENDIX F

HIGH SPEED SWEEP CIRCUIT FOR PHASE AND WIDTH DISPLAY

F.1. The sweep circuits in general use are limited to repetition frequencies well below one megacycle. For the phase and width display we require a maximum repetition frequency of 10 megacycles. This must be locked to a voltage source with a reasonable phase error at all frequencies. There must be no discontinuities or "kinks" (so that small amplitude phase oscillations may be observed) but high accuracy is not required.

F.2. Because of the high frequency, a sweep generator which depends upon amplification of an irregular wave form cannot be considered. In particular, feedback circuits are ruled out because of delays, which, though they do not prevent amplification, cause a large amount of phase shift in the feedback path. Miller circuits[29] of all forms are out of the question because of these considerations. The synchronization requirement makes it advisable to keep the circuit extremely simple so that phase shifts will not vary with frequency. Finally, it would be desirable to generate the sweep at a level high enough to avoid amplification. Amplification is extremely difficult, especially if the flyback time is short. A divider should not be considered, so that the flyback time must come out of the 360 degrees in which a display is possible. The flyback time should therefore be as short as possible.

SAWTOOTH WAVE

F.3. The sawtooth wave, $V = F(\Theta)$ is an odd function when disposed about the axis shown (figure F.3).

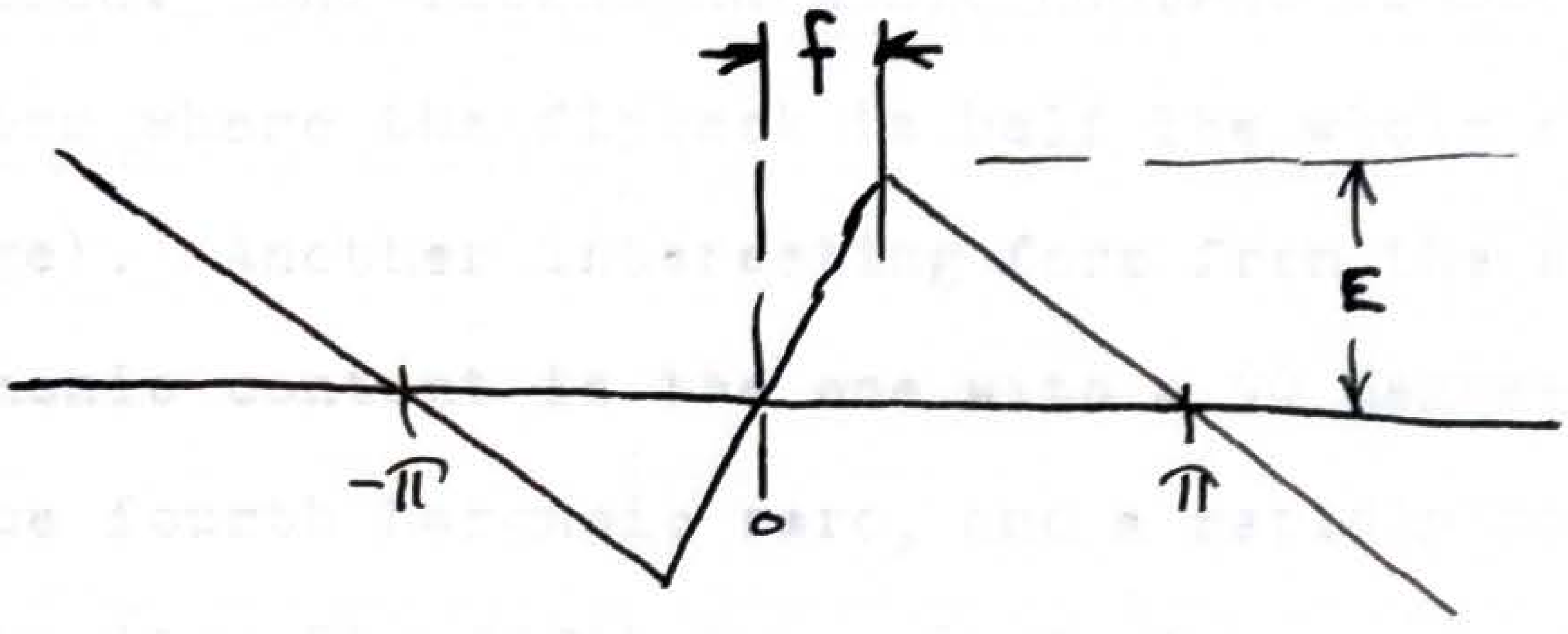


Figure F.3

$$F(\theta) = A_1 \sin \theta + A_2 \sin 2\theta + \ldots + A_n \sin n\theta \qquad (F.3.1)$$

$$A_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} F(\theta) \sin n\theta \, d\theta \qquad (F.3.2)$$

$$= 2E \frac{1}{f(\pi - f)} \frac{\sin nf}{n^2}$$

where f is the half length of the flyback in radians. When f approaches zero, the amplitude function approaches

$$\frac{2E}{\pi} \cdot \frac{1}{n}$$

When the flyback is half the whole cycle, $f = \frac{\pi}{2}$,

$$A_n = \frac{8E}{\pi^2} \cdot \frac{1}{n^2} \left[\sin \frac{n\pi}{2}\right] .$$

The terms decrease as $1/n^2$ and all the even terms are missing. When f = 0 the initial slope attained by including all the harmonics up to the m^{th} is:

$$\tan \theta = \frac{2E}{\pi} m .$$

F.4. The limiting form when the flyback is infinitely short ($f = 0$) can be seen to contain the greatest proportion of high harmonics. The lowest harmonic content is shown in the limiting form where the flyback is half the whole cycle (triangular wave). Another interesting form from the standpoint of low harmonic content is the one with a 90 degree flyback. This has the fourth harmnoic zero, and a rapidly decreasing series up to it. The fifth harmonic is 4 per cent of the fundamental. The result obtained by using 3 harmonics only shows the usual rounding of sharp corners accompanied by "waves" on the straight part, the "waves" having the period of the last harmonic. In connection with the reproduction of a wave form requiring high harmonics, it should be noted that while pre-accentuation of high frequencies is possible where the amplifier characteristic unavoidably droops, it is likely to be impractical over the frequency range considered.

CIRCUIT SCALING

F.5. It was pointed out in Appendix A that with modern miniature tubes in which the leads are very short and the transit time small, the important elements of a circuit are often resistances and capacities. The position and value of these elements are usually known with certainty. This suggests that a circuit may be "scaled down" merely by increasing all capacities and decreasing the frequency in the same proportion. This device was extremely useful in testing and developing

sweep circuits, filters, and components to be used at 10 megacycles. In the scaled-down version, a thorough analysis could be made using conventional oscilloscopes, etc. The number of tests, necessarily difficult, at 10 megacycles would then be greatly reduced.

CIRCUITS

F.6. The unavoidable capacity in the circuit is that of the cathode ray tube. If this capacity is charged at the rate required for the fastest sweep (with constant current) a tube is required only when the capacity has to be discharged at the end of the sweep. A rough comparison between this way of using a tube and its use as an amplifier shows that a very great economy results from the former. The capacity of a cathode ray tube (Cossor) plus that of an 807 amplifier tube is 25 micromicrofarads. The voltage required for one full sweep is 110 volts. The current for constant current charging is 27 milliamperes. As a conventional amplifier a change of about 500 millamperes is required to amplify the first three harmonics (up to 30 megacycles). The average current is half this or 250 milliamperes. Since this cannot be obtained from one tube, the current actually required is much greater, since one is approaching the limiting condition where the tube is working into its own output capacity.

F.7. If the tube is required to return the sweep only, the average current is 27 milliamperes (for short flybacks). If the flyback is 1/10, the peak current is 10 times this

which is quite within the capabilities of a tube such as the 807. In addition to the current reduction, a much more rapid flyback is obtained, and the departures from linearity due to insufficient harmonics are avoided. The departures from linearity encountered when charging a capacity through a fixed resistor from a limited voltage supply are still present but the error so arising can be made negligible. The limit on the whole phase and width display is set by the definition of beam shape which it is possible to achieve with the signal amplifier.

F.8. Because of the high voltage available at the synchrotron accelerating electrode, the flyback may be controlled by applying a large voltage to the grid of the sweep tube and arranging the coupling constants so that plate current flows only for a small time at the "top" of the applied wave. The choice of the 807 as the sweep tube was primarily because of its good pulse characteristics. It is necessary also that the discharge of the capacity take place on the "non-pentode" part of the plate characteristics, that is, the region where the current is a function of the plate voltage. This is so that automatic adjustment to the current required will take place with a small change of voltage. The grid circuit combination is chosen to give the desired flyback time and the desired rate of adjustment to possible changes in the applied R.F.

F.9. With the cathode ray tube used, the limit to the shortness of the flyback time was actually the high frequency resonances of the cathode ray tube. The cathode ray tube to

be used finally has much lower capacities, and it is hoped that the resonances will be correspondingly higher and less troublesome.

AMPLITUDE CONTROL

F.10. Since the length of the sweep depends upon frequency, a method of keeping this constant is necessary. A satisfactory amplitude control circuit can be arranged by adjusting the charging voltage through a feedback loop.

APPENDIX G

AMPLIFIER FOR PHASE AND WIDTH DISPLAY

G.1. For a band extending to 30 megacycles, one of the standard video amplifier circuits [30] may be shown to be capable of providing sufficient constancy of gain and delay. The

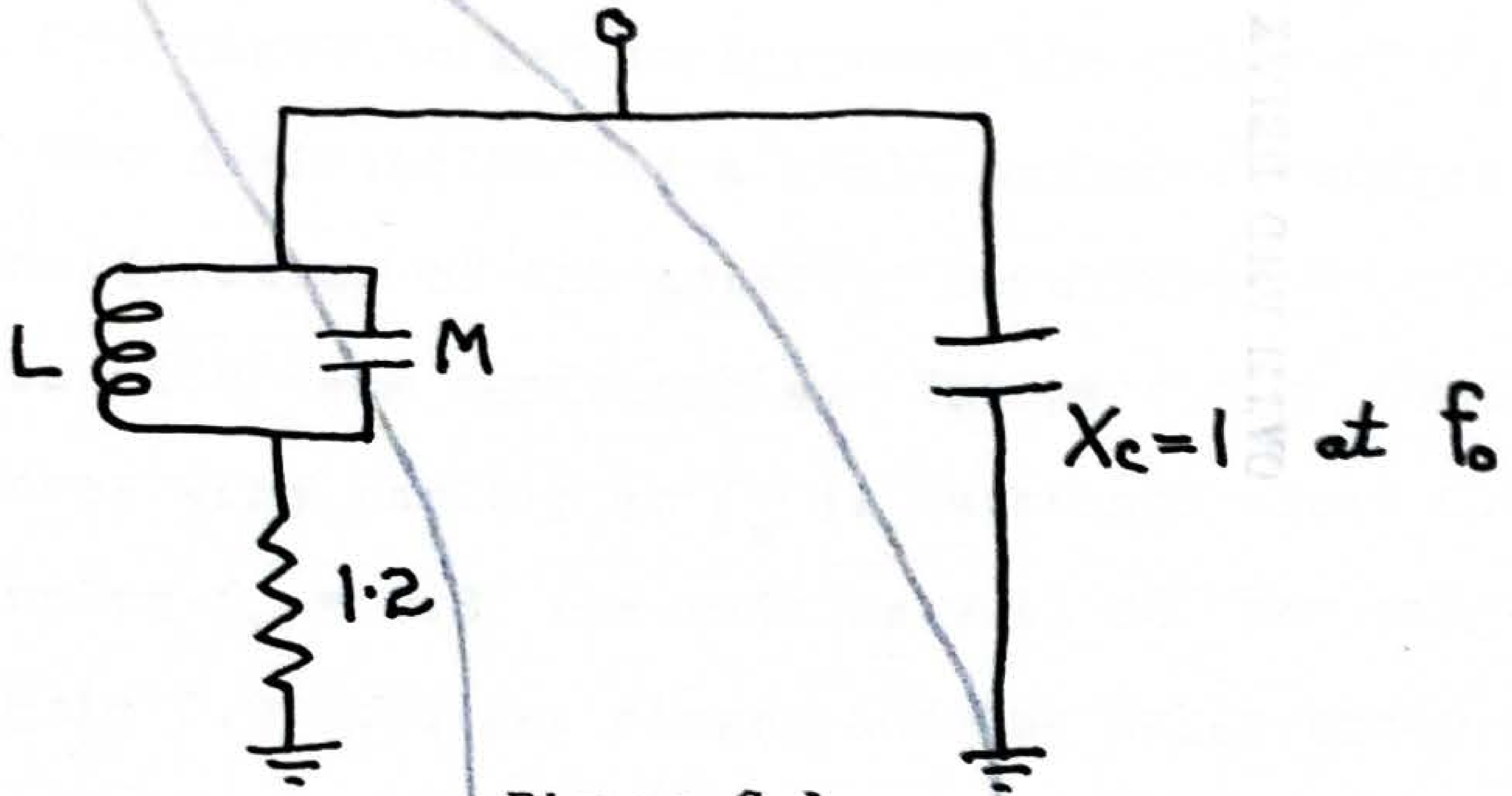


Figure G.1

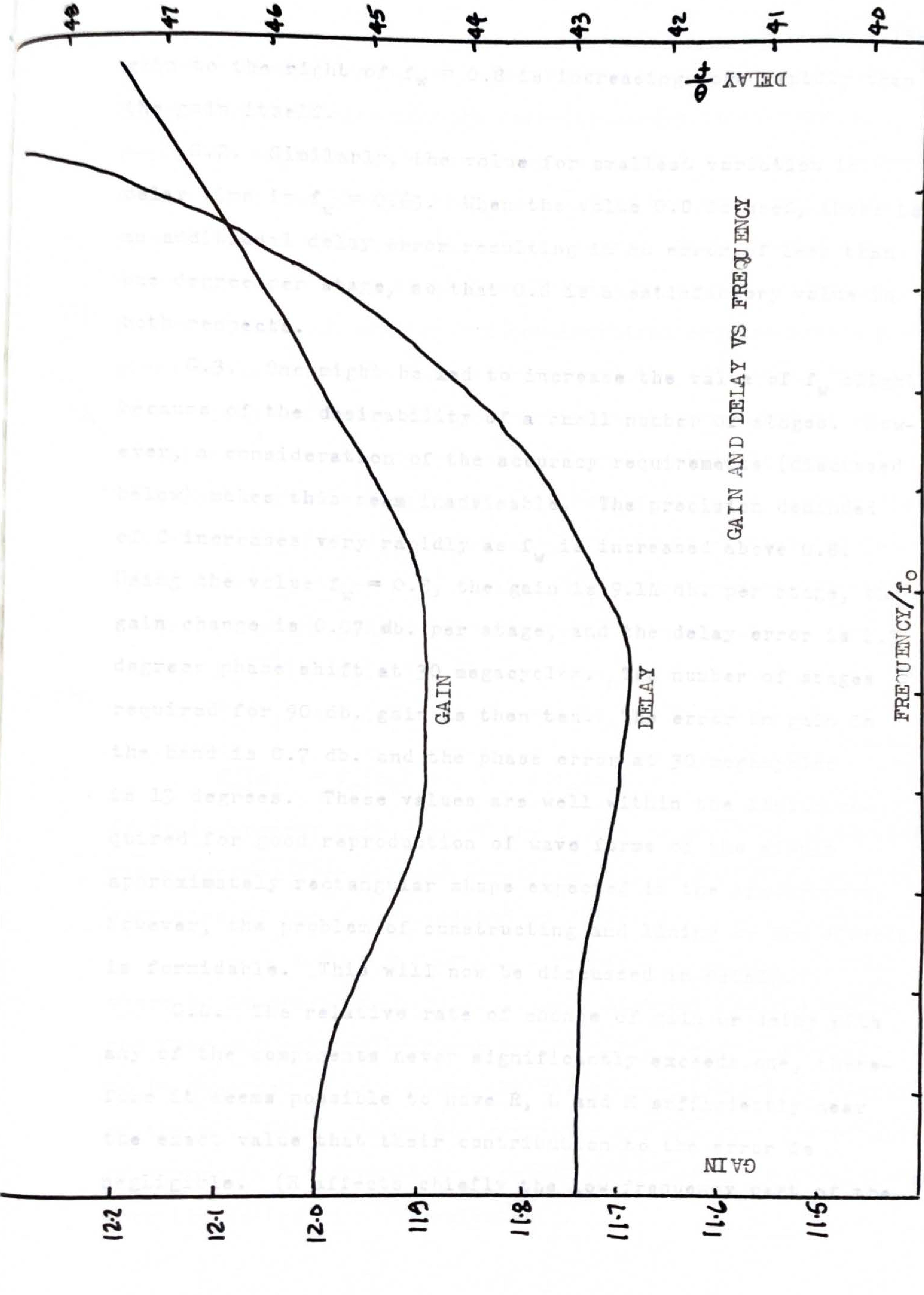
computed characteristics are shown in the following graph. For the EF 91, when $0.7f_o = 30$ megacycles, R = 389 ohms, and the gain is 9.14 db. The exact expression for gain and change of gain (maximum variation in the band) as a function of f_o is too difficult to handle, but a sufficient approximation for the present purpose is

$$\text{Gain in db.} = (9.14 + 20 \log \frac{f_w}{0.8})n \qquad \text{(G.1.1)}$$

$$\text{Gain change in db.} = (0.4f_w - 0.25)n, \quad f_w \geq 0.8 \qquad \text{(G.2.1)}$$

Here $f_w = 30/f_o$ and n = number of stages. The best value of f_w may be seen to be at least 0.8, since the maximum variation is constant up to this value, excluding very low values of gain. From the above relations it may also be shown that the best value is not greater than 0 .8 since the change of

Figure G.1


GAIN AND DELAY VS FREQUENCY
DELAY
GAIN
DELAY
GAIN
FREQUENCY/f_o
48
47
46
45
44
43
42
41
40
12.2
12.1
12.0
11.9
11.8
11.7
11.6
11.5
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0

gain to the right of $f_w = 0.8$ is increasing more rapidly than the gain itself.

G.2. Similarly, the value for smallest variation in delay time is $f_w = 0.65$. When the value 0.8 is used, there is an additional delay error resulting in an error of less than one degree per stage, so that 0.8 is a satisfactory value in both respects.

G.3. One might be led to increase the value of f_w slightly because of the desirability of a small number of stages. However, a consideration of the accuracy requirements (discussed below) makes this seem inadvisable. The precision demanded of C increases very rapidly as f_w is increased above 0.8. Using the value $f_w = 0.8$, the gain is 9.14 db. per stage, the gain change is 0.07 db. per stage, and the delay error is 1.3 degrees phase shift at 30 megacycles. The number of stages required for 90 db. gain is then ten. The error in gain in the band is 0.7 db. and the phase error at 30 megacycles is 13 degrees. These values are well within the limits required for good reproduction of wave forms of the simple approximately rectangular shape expected in the synchrotron. However, the problem of constructing and lining up the circuit is formidable. This will now be discussed in detail.

G.4. The relative rate of change of gain or delay with any of the components never significantly exceeds one, therefore it seems possible to have R, L and M sufficiently near the exact value that their contribution to the error is negligible. (R affects chiefly the low frequency part of the

response curve, L the middle, and M the high frequency part.) Since slight changes in tube capacity cannot be avoided, C cannot be regarded as under accurate control. One can, however, assume that the changes that occur after the tubes have been properly aged will be of a random nature. It is therefore necessary to be sure that these changes in C will tend to cancel in the result. A similar but not identical requirement is that, given small errors in C in some stages, the effect at all points of the gain and delay characteristic may be corrected by an adjustment made on other stages. This is desirable since it will be impossible in general to find which stages contain the deviations. If $f_i(a_j)$ is the error in the characteristic produced at a frequency i by an error a_j in the capacity C_j of the condenser in the jth stage, then we have:

$$\begin{aligned} f_1(a_1) + f_1(a_2) + \cdots + f_1(a_n) &= 0 \\ f_2(a_1) + f_2(a_2) + \cdots + f_2(a_n) &= 0 \\ \cdot \quad\quad \cdot \quad\quad \cdots \quad\quad \cdot \quad & \\ f_m(a_1) + f_m(a_2) + \cdots + f_m(a_n) &= 0 \end{aligned} \tag{G.4.1}$$

If it is assumed that some of the C_j may be adjusted to satisfy one equation, then all the equations will be satisfied at the same time if the f_i are related as follows:

$$k_1 f_1 = k_2 f_2 = \cdots = k_m f_m \tag{G.4.2}$$

where k_i is constant, $i = 1,2,\ldots,m$. Although the f's may be of any form subject to the above restriction, the only case likely to be realized is that where the f's approximate linear functions of the a_j. This is also the condition for a normal distribution of errors in C to result in a normal distribution of deviations from the design gain characteristic.

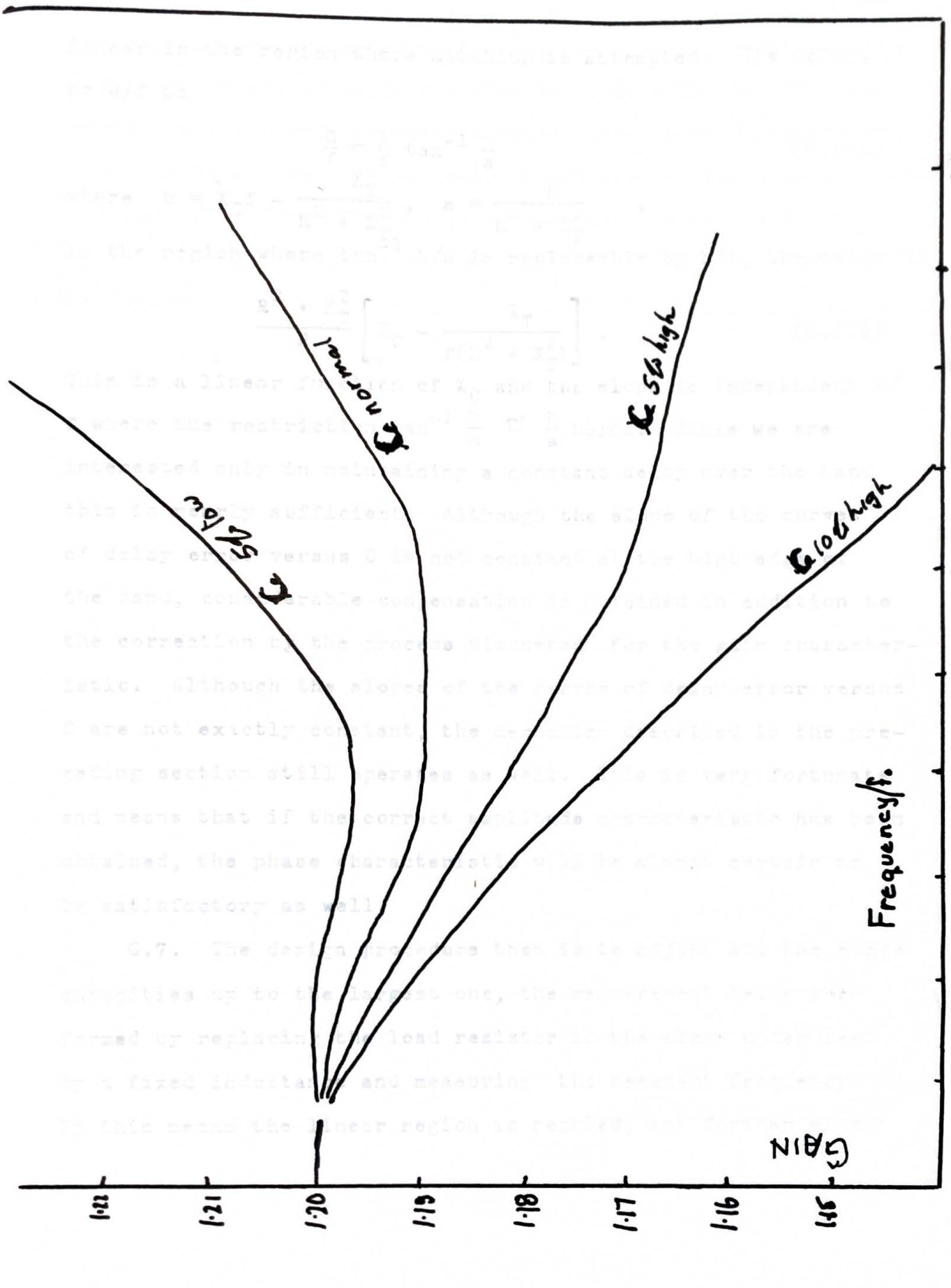
G.5. Thus the initial lining up procedure must ensure that all the stages are close enough to the design stage that the linear relation holds at all points. Random adjustment using a swept oscillator would not be satisfactory. After the values have been located within the safe limits, however, a certain amount of final adjustment is permissible, since one can now be sure that the form of the characteristic if correct at one point will be correct at all others. The characteristic curves obtained during this final adjustment will be one of the family indicated in the following graph, figure G.5. The gain as a function of X_C is

$$\frac{1}{\sqrt{\left(\frac{R}{R^2 - X_T^2}\right)^2 + \left(X_C - f - \frac{X_T}{R^2 + X_T^2}\right)^2}} \qquad (G.5.1)$$

where X_T is the impedance of the tuned circuit at frequency f. (See figure G.1). To see how far the linear region extends, the variation in gain as a function of the error in C has been plotted for f = 0.8 (the high edge of the band), f = 0.6, and f = 0.4, figure G.5. The curves show that the linear region is of appreciable extent. It should be possible to get C within the limits indicated, when the design characteristic may be approached by observing how it changes with C.

G.6. The condition required to enable the time delay to be matched to the design curve is the same as discussed for the amplitude. That is, in practice, the relation between departure from the design curve and error in X_C must be

Figure G.5



GAIN
Frequency/f.
1·22
1·21
1·20
1·19
1·18
1·17
1·16
1·15
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
C 5% low
C normal
C 5% high
C 10% high

linear in the region where matching is attempted. The delay, or θ/f is

$$\frac{\theta}{f} = \frac{1}{f}\tan^{-1}\frac{b}{a} \tag{G.6.1}$$

where $b = X_C f - \dfrac{X_T}{R^2 + X_{T_1}^2}$, $a = \dfrac{R}{R^2 + X_T^2}$.

In the region where $\tan^{-1} b/a$ is replaceable by b/a, the delay is

$$\frac{R^2 + X_T^2}{R}\left[X_C - \frac{X_T}{f(R^2 + X_T^2)}\right] . \tag{G.6.2}$$

This is a linear function of X_C and the slope is independent of f where the restriction $\tan^{-1}\frac{b}{a} \simeq \frac{b}{a}$ holds. Since we are interested only in maintaining a constant delay over the band this is nearly sufficient. Although the slope of the curves of delay error versus C is not constant at the high edge of the band, considerable compensation is obtained in addition to the correction by the process discussed for the gain characteristic. Although the slopes of the curves of delay error versus C are not exactly constant, the mechanism described in the preceding section still operates as well. This is very fortunate and means that if the correct amplitude characteristic has been obtained, the phase characteristic will be almost certain to be satisfactory as well.

G.7. The design procedure then is to adjust all the stage capacities up to the largest one, the measurement being performed by replacing the load resistor in the stage under test by a fixed inductance and measuring the resonant frequency. By this means the linear region is reached, and further slight

adjustments are indicated by the amplitude characteristic. For A.V.C. adjustments of gain, a number of tubes with very low load resistors and simple inductance compensation have to be inserted down the line, these tubes being in addition to the ones required for amplification. The change in capacity due to the A.V.C. action then does not seriously alter the gain or delay characteristics.

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